MACROECONOMICS COMPREHENSIVE EXAM

January 2015

This exam consists of three Questions with equal weights.
Question I

Question I involves 2 problems, each of which have some choice on which parts to do. The first problem gets a weight of 20 percent, the second problem, 80 percent. All parts are of equal value.

1. For problem 1, do 3 of 5 parts

(a) Nominal debt-to-GDP ratios for major economies at present fall approximately in what range?

(b) The nominal interest rates on, say, 10-year government bonds of major economies fall approximately in what range?

(c) Labor's share of income in the U.S. has been very steady since the start of the Great Moderation. [True/False and discuss]

(d) Why is deflation (negative inflation rates) considered dangerous or harmful?

(e) Discuss the merits of the representative agent model for explaining aggregate consumption.
2. For problem 2, do all parts except where otherwise specified.

(a) The standard approach to intertemporal optimization, consumers use borrowing and saving to smooth consumption in response to ups and downs in income. This phenomenon is reflected in the permanent income hypothesis (PIH) and under certain assumptions implies that consumption should approximately follow a random walk.

**Problem:** Describe the PIH and describe a simple model in which it holds. Do the same for the random walk hypothesis.

(b) In a model with government expenditures that must be paid for with tax revenues, governments can use borrowing and saving to smooth the path of taxes in the face of variation in expenditures. Under certain conditions, however, Ricardian equivalence will hold and, for a given path of government expenditures, the consumption and welfare of agents will be invariant to how the government happens to rearrange the timing of taxes through the use of borrowing.

**Problem:** Explain Ricardian equivalence and some key conditions under which it holds.

(c) If the only tax instruments available to the government are distorting taxes, Ricardian equivalence will not generally hold. (Hint: if you did not include this issue in the previous answer you might want to re-consider.) Under some standard conditions, if taxes are distorting, a welfare maximizing government has an incentive to smooth tax revenues in order to minimize distortions.

**Problem:** Explain why we might think that the total welfare losses due to tax distortions would rise with the variability marginal tax rates, even under certainty over the path of tax rates.
(d) Tax smoothing motives can give rise to results analogous to the PIH and random walk results discussed above. Explore these in a simple, partial equilibrium structure with all real variables.

In particular, assume that the path of government expenditures is exogenous and that the government maximizes welfare by choosing taxes to minimize:

\[ -\frac{1}{2} E \left[ \sum_{t=0}^{\infty} \beta^t \tau_t^2 \right] \]

where \( \beta \) is a discount factor and \( \tau \) is real tax revenues.\(^2\)

The government budget constraint is:

\[ b_t = \rho b_{t-1} + g_t - \tau_t \quad \text{for all} \ t \]

where \( b_t \) is the real value of period \( t \) government debt and \( g_t \) is real government expenditure (exogenous), and \( \rho \) is the gross real rate of return, which is assumed to be constant. Take the simple case in which \( \rho \beta = 1 \).

**Problem:** Fill in an argument that optimal taxes will follow a random walk in this model stating conditions as needed. That is, show:

\[ E_t \tau_{t+1} = \tau_t \]

(e) **Problem:** Make an argument that the real value of government debt in period \( t \), \( b_t \), should equal the expected present value of future government surpluses—that is, the present of surpluses from \( t + 1 \) onward.

(f) **Problem:** Assuming that taxes follow a random walk, write an expression for the expected present value at time \( t \) of government

\(^1\)Note: we could derive this loss function starting from the consumers problem.

\(^2\)Assuming, for example, that the tax is an income tax and that consumer income is exogenous and constant, we can interchangeably talk about tax revenue and the marginal tax rate.
surpluses from $t+1$ onward and show that this implies that the value of debt follows a random walk.

(g) For this part, do 1 of these 2

i. So far, we have discussed a one-period, risk free, bond the pay-off of which is not state contingent. Suppose instead that the government could issue bonds in any period, $t$, that payoff in any future period $t+s$ as a function of government spending in the payoff period, $g_{t+s}$.

**Problem:** Make an argument that with this contingent debt, optimal taxes will be constant.

ii. So far, we have a real model. Suppose we changed to a nominal model. The path of real government expenditures remains exogenous and the distorting effects of taxes remain stated in terms of the real value of taxes. But the government bonds pay off in money units and taxes are collected in money units.

**Problem:** Explain how surprise inflation works as a tax in this model and discuss the merits of substituting variable inflation for tax variability in light of the smoothing results above. (To keep from getting too far into monetary economics, you may simply suppose that the government can set the price level at the beginning of any period.)
Question II

II.1) [25] Short Questions
Answer the following questions in a few paragraphs each including some algebra:

a) [5] Derive the IS curve in the IS/LM framework and explain why it is downward sloping.

b) [5] State and interpret the dynamic IS (DIS) curve in a typical New Keynesian model.

c) [5] Point out similarities and differences between the IS curve and DIS curve and the economic mechanisms that they capture.

d) [5] Describe the no-shirking condition in the Shapiro-Stiglitz efficiency wage model and explain the comparative statics it gives rise to.

e) [5] Taking a step back, what are efficiency wage models good for, and what other efficiency wage models are there? What are matching models of the labor market good for?

II.2) [50] Financial Constraints and Externalities
Assume an economy with three time periods \( t = 0, 1, 2 \) in which there are two sets of atomistic risk-neutral private agents, entrepreneurs \( e \) and households \( h \), of unit mass each. Households have large endowments \( w \) every period; entrepreneurs have no endowments but they have an investment opportunity.

In period 0, entrepreneurs can create \( k \) units of capital goods at cost \( J(k) \) consumption goods, which is given by an increasing strictly convex function that satisfies \( J(0) = J'(0) = 0 \) and \( J'', J''' > 0 \) for \( k > 0 \). They borrow \( b \) from households using non-contingent one period debt.

In period 1, entrepreneurs earn a stochastic income \( \bar{A}k \) from their capital with \( \bar{A} \in [A_{\text{min}}, A_{\text{max}}] \subset \mathbb{R}^+ \) and need to repay their debt. To capture financial constraints in a simple yet tractable manner, let us make the extreme assumption that entrepreneurs are not constrained in their borrowing in period 0 but cannot borrow at all in period 1. Although they cannot borrow in period 1, they can sell capital goods to households at a market price of \( q \). Observe that the consumption of all agents needs to be non-negative in every period.

In period 2, both sets of agents produce consumption goods using their capital. The production technology of entrepreneurs is \( \bar{A}k_2^2 \), that of households is \( \bar{A}\log(1 + k_2^2) \). Both sets of agents consume their production and endowment, and capital fully depreciates.

a) State the utility functions and budget constraints of both sets of agents.

b) Explain the point behind each of the assumptions on the function \( J(k) \).

c) Write down the optimization problem of each agent as well as all market clearing conditions.

d) Now let’s proceed by backward induction. What are the state variables of the economy in period 2, and what is the utility of each type of agent as a function of these state variables?
At the beginning of period 1, the state of the economy is fully described by capital $k$ and by the financial net worth $n^e = Ak - b$ of entrepreneurs and $n^h = w + b$ of households. When an individual agent solves her optimization problem, she takes the aggregate capital and sector-wide net worth of the rest of the economy as given. To distinguish between the individual-level variables and aggregate/sector-wide variables, we denote individual variables by lowercase letters $k$, $n^e$, $n^h$ and aggregate/sector-wide variables by capital letters $K$, $N^e$, $N^h$. In equilibrium, the two are of course identical.

The period 1 value function of an individual entrepreneur is given by $V^e (n^e, k; N^e)$ and depends only on the entrepreneur's individual variables and sector-wide entrepreneurial net worth $N^e$. Similarly, the value function of an individual households is given by $V^h (n^h; N^e)$.

e) Let us focus on households at the beginning of period 1. Solve $\max V^h (\cdot)$ subject to the relevant constraints and describe the optimality condition that characterizes the household demand for capital. Why did we include $N^e$ as an argument in the value function of households? What is the market price of capital if $k_2^h = 0$ or if $k_2^h = 0$? What is the maximum revenue that entrepreneurs can obtain if they sell all their capital to households? Can you obtain an explicit expression for $V^h (\cdot)$?

f) Let us next focus on entrepreneurs at the beginning of period 1. Solve $\max V^e (\cdot)$ subject to the relevant constraints, where an individual entrepreneur takes $N^e$ as given. Describe the threshold level of productivity $n^e$ below which entrepreneurs start to sell assets. In a symmetric equilibrium with $n^e = N^e$, describe the amount of sales and the level of entrepreneur consumption as a function of $N^e$. What is the lowest possible level of $N^e$ for which the problem admits a solution? Can you obtain an explicit expression for $V^e (\cdot)$? In the following, assume that $A_{\min}$ is sufficiently high that $N^e$ is always above this level in equilibrium.

g) Now let us focus on period 0. Denote the optimization problem of both types of private agents $i = e, h$ employing the continuation utility given by the value function $V^i (\cdot)$. State the optimality conditions that determine optimal borrowing, lending, and investment.

h) Given the privately optimal choice of $k$, will entrepreneurs find it necessary to sell capital in the lowest state $A_{\min}$ of period 1?

i) Now compare the period 0 optimization problem of private agents with the problem of a constrained social planner who chooses the period 0 allocations in the economy and takes the continuation utility $V^i (\cdot)$ of both sets of agents as given. Note that the constrained planner internalizes that individual variables and sector-wide variables are identical in equilibrium, i.e. $n^e = N^e$.
Question III

Consider the textbook Solow growth model. Let $s$ be the saving rate, $n$ the population growth rate, and $d$ the depreciation rate. Assume the aggregate production function is $Y = F(K, EL)$, where the $E$ is the efficiency of labor, which has a growth rate of $g$. Unless otherwise noted, the production function exhibits constant returns to scale, and capital and labor are paid their marginal products.

A. Studies of the U.S. economy often assume the production function is Cobb-Douglas: $Y = K^a (EL)^{1-a}$. What facts about the economy suggest that this assumption is realistic? What do economists believe is the approximate value of the parameter $a$, and why?

B. For the Cobb-Douglas production function in Part A, derive the steady state level of output per effective worker, $Y/(EL)$, as a function of the model’s parameters.

C. Suppose the economy starts in steady state, and the growth rate $g$ suddenly increases. In graphs, show the paths over time of output ($Y$), output per worker ($Y/L$), and output per effective worker ($Y/(EL)$). The graphs should have time on the horizontal axis, and cover a period beginning in the initial steady state and ending after the economy has returned to steady state with the higher $g$.

D. In the Solow model, what does it mean for the economy to be dynamically efficient? Give an economic explanation of why dynamic inefficiency is possible. Write a condition that determines whether the economy is dynamically efficient, and briefly explain why this condition makes sense (formal derivation is not necessary).

E. Suppose the economy begins in steady state and the government introduces a policy of redistributing income: saving is taxed at a rate $\tau$, with the proceeds given to workers. Investment equals after-tax saving, so it is a fraction $s(1 - \tau)$ of output. Let the “net wage” be the sum of the usual wage (the marginal product of labor) and the per-capita subsidy to workers (the total proceeds from the tax divided by L). In this part, you can assume for simplicity that the production function is Cobb-Douglas, and the efficiency of labor $E$ is constant.

(i) Derive the net wage immediately after the new policy is introduced as a function of $\tau$ and the other parameters. What value of $\tau$ maximizes the net wage at this point (assuming $\tau$ must lie between zero and one)? Derive the answer mathematically and provide economic intuition.

(ii) Derive the steady state level of the net wage after the new policy is introduced. What value of $\tau$ maximizes the steady state net wage (assuming $\tau$ must lie between zero and one)? Derive the answer mathematically and provide economic intuition.
F. Now assume the aggregate production function is $Y = K^\alpha (EL)^\beta$, where $0 < \alpha + \beta < 1$. Otherwise, make the usual assumptions in the Solow model. Discuss how the process of economic growth in this case differs from the standard Solow results. Will per capita income rise, fall, or stay constant over time? Are different answers possible depending on parameter values? Try to give precise answers, and provide economic intuition (formal derivations not required).