Johns Hopkins University
Department of Economics
Microeconomics comprehensive examination
August 18, 2014

Instructions

- You have 220 minutes for the exams plus 15 additional minutes to read it.
- Please answer all questions. The amount of time you need to answer each question is given in parenthesis. The questions are weighted by the amount of time you need to answer them. Parts A and B are 80 points each, and Part C is 60 points.
- PLEASE START EACH QUESTION ON A NEW PAGE.
- PLEASE MAKE SURE TO DEFINE YOUR TERMS PRECISELY INCLUDING ANY NOTATIONS THAT YOU INTRODUCE

Part A

1. (40 min) Each of the following statements is valid or invalid. If valid, provide an argument for its validity; and if false, a counter-argument. These can involve a proof or a counterexample. A well-labeled diagram can serve as a counterexample, and clear English sentences can go far in delineating a proof.

(a) Since the assumption of constant returns to scale translates into a production set that is a cone, it guarantees the convexity hypothesis on the production set.

(b) If a production set is convex and closed, then each of its boundary points is a technologically efficient production plan.

(c) The neoclassical theory of the consumer shows that in a finite-commodity world, each commodity must, of necessity, be strictly decreasing in terms of its price.

(d) Every monotonic preference relation over $\mathbb{R}_+^n \times \mathbb{R}_+^n$ is complete.

(e) Since every upper semicontinuous convex-valued correspondence has a continuous selection, the Kakutani fixed-point theorem is a trivial consequence of the Brouwer fixed point theorem.

(f) The set of maximizers of an objective function over a constraint set is an upper semi-continuous correspondence over a the set of parameters if the constraint set is an upper semi-continuous correspondence over the parameter set and the objective function is jointly continuous over the set of parameters and the variable that is being maximized.
(g) A core allocation of a finite exchange economy exhibits the “equal treatment property” because it is a subset of a Pareto optimal allocation of the economy.

2. (40 min) Consider an exchange economy with two goods and two consumers, A and B. Consumer A likes both commodities equally and has tastes parametrized by linear indifference curves $x + y$. The preferences of consumer B are given by $\min[x, y]$ and thus parametrized by kinked [Leontief] indifference curves. Consumer A’s initial endowment is (6, 2) and consumer B’s initial endowment is (4, 6). For this situation:

(i) Draw the Edgeworth box for this economy, taking care to label representative indifference curves for each consumer.

(ii) Label the set of individually rational, Pareto optimal and the core allocations.

(iii) Explain in words why giving all of the economy’s resources to A is a Pareto optimal allocation but giving them to B is not.

(iv) Given (iii) above, should a “reasonable” social welfare function weigh agent A higher than agent B? Explain.

(v) Determine a competitive equilibrium for this economy.

(vi) Explain clearly what features of this numerical example do not carry over to the general theory as developed not only in Debreu’s *Theory of Value*, but also embracing questions concerned with cardinality of competitive equilibria and the relationship between core and competitive allocations.
Part B

3. (40 min)

Consider a two period game between two players. In the first period, the two players play the prisoner's dilemma

\[
\begin{array}{cc}
C & D \\
3,3 & 0,4 \\
4,0 & 1,1 \\
\end{array}
\]

After observing the outcome in the first period, the two players play the coordination game

\[
\begin{array}{cc}
C & D \\
3,3 & 0,0 \\
0,0 & 1,1 \\
\end{array}
\]

The payoff of each player is the sum of the payoffs from two periods.

(a) What is the set of pure strategies for player 1?
(b) Define pure strategy subgame perfect equilibrium for this game.
(c) Construct a subgame perfect equilibrium in which each player chooses C in the first period.

4. (40 min)

Consider the following game:

\[
\begin{align*}
A \rightarrow B & \rightarrow 2 \rightarrow D \rightarrow 3 \rightarrow F \rightarrow (8,-1,1) \\
& \leftarrow C \leftarrow E \\
(2,3,3) & \rightarrow L \rightarrow R \rightarrow L \rightarrow R \\
(1,0,0) & \rightarrow (0,1,0) \rightarrow (0,1,2) \rightarrow (1,2,0)
\end{align*}
\]

(a) Define weak sequential equilibrium for this game
(b) Find the weak sequential equilibria.
Part C

5. (20 min) Consider a agent whose preference relation, \( \succeq \), on the set \( D_J \) of cumulative distribution functions (c.d.f) on a compact interval, \( J \), is continuous and satisfies the independence axiom. Moreover, suppose that, for all \( F, G \in D_J \), \( F \succeq G \) if and only if \( F \) dominates \( G \) according to first-order stochastic dominance.

(a) Define the terms c.d.f and \( F \) dominates \( G \) according to first-order stochastic dominance.

(b) Is the preference relation \( \succeq \) a weak order? Prove your assertion.

(c) What is the representation of \( \succeq ? \)

6. (40 min) Consider a competitive insurance market in which the riskiness of agents may be modified by taking risk-reducing measures, and that taking these measures costs money. The agents are identical except for the cost of taking the risk reducing measures. Assume that the agents are risk averse, have initial wealth \( w \), and they are exposed to the risk of losing the amount \( x \), \( 0 < x < w \). Assume that there are two levels of risk, \( p_1 < p_0 \), where \( p_1 \) denotes the probability of loss if the risk-reducing measures are implemented and \( p_0 \) denotes the probability of loss if they are not.

Let there be two types of agents: Agents of type \( A \) can achieve the risk reduction at the cost \( c_A \), and agents of type \( B \) can achieve the risk reduction at the cost \( c_B \). Suppose further that the agent's type is private information, that \( c_B > c_A > 0 \) and that if no risk reduction measures are taken then the cost is zero.

The insurers in this market are expected profit maximizers and compete by offering insurance policies.

(a) Define an equilibrium for this market (hint: think of a modified version of the Rothchild-Stiglitz screening equilibrium).

(b) Describe to the best of your ability the nature of possible equilibria in this market (that is, what are the equilibrium contracts).