JOHNS HOPKINS UNIVERSITY  
Department of Economics  
Microeconomics Comprehensive Examination  
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Answer all questions. In your answers, please make sure to define your terms and notations precisely. You have four hours. The estimated time recommended for each question is given in parenthesis and the relative amount of time roughly corresponds to the weight of each question in the final grade. PLEASE START ANSWERING EACH QUESTION ON A NEW PAGE. Good luck.

Part I

1. (40 minutes) This question is in two parts: the first concerns some basic results in microeconomic theory, and the second covers material discussed in the lectures. Sharp and succinct answers are preferable to verbose expositions of irrelevant material.

   (a) The questions in this part do not require extensive writing; clearly-labeled diagrams should carry you a considerable way.

   (i) Consider an output $Y$ being produced by capital $K$ and labor $L$ according to the specification $Y = F(K, L)$. Depict this relationship in three two-dimensional diagrams, specifying in each diagram what the slopes of the relevant curves refer to. Use the intuition so obtained to justify the use of the notational convention for the rates of substitution for a technology pertaining to an arbitrary finite number of outputs and inputs. Explain the implication of the convexity postulate on the production set for such rates of substitution.

   (ii) Let $f : \mathbb{R}_+ \rightarrow \mathbb{R}_+$ be a differentiable function from the set of non-negative real numbers to itself. Let $x^*$ be a maximum. Write down the necessary (first-order) conditions that $x^*$ must satisfy.

   (iii) Consider a profit-maximizing firm with the technology specified in (i) above, and which is too small to influence the prices of its three commodities. Write down the necessary (first-order) conditions pertaining to the amounts which this firm buys from, and sells to, the market. Do these amounts necessarily have to be unique? If the price of one of the commodities in question rises, what would be the effect on its equilibrium level? Suggest a derivation of your result that does not depend on the necessary conditions.
(iv) Under the additional condition that \( F(\cdot, \cdot) \) exhibits constant returns to scale, depict the relationship in another two-dimensional diagram, again specifying the meaning of the slope of the relevant curve. Briefly sketch how this simplification can be used to specify a Solow-Swan type aggregative growth model. In not more that two-sentences, show how the Solow specification differs from that of a Uzawa type two-sector growth model.

(b) Explain concisely the differences between the concepts and results listed under each part.

(i) The Rybczynski and reciprocity theorems in the two-sector model of general economic equilibrium.

(ii) The duality and equilibrium theorems of linear programming.

(iii) The convexity of the constraint set, the Slater's form of the constraint qualification and the concavity of the functions specifying the constraint; all of these in the context of the theory of finite-dimensional constrained optimization.

(iv) Pontryagin's maximum principle and the Euler-Lagrange conditions in the calculus of variations.

2. (40 minutes) Consider a two-person, two-commodity economy in which the first person likes the second commodity twice as much as the first, and the second person half as much. Each person has only a unit amount of the one commodity that he or she does not prefer to the other, both commodities being universally desirable.

(i) Depict this economy in an Edgeworth box diagram, specifying clearly the preferences of each person and the initial endowments.

(ii) Consider a price system in which the prices of the two commodities are not identical. For such a price system, exhibit the budget set that is faced by each person. Delineate the role that the aggregate endowments play in the determination of these budget sets.

(iii) Depict the set of competitive equilibria of such an economy. In intuition expressed in clear English sentences, explain why this set is, or is not, a singleton set.

(iv) Consider an allocation that is interior to the box, which is to say each person receives a positive amount of each commodity. Is such an allocation Pareto optimal? Justify your answer by providing both a geometric and a verbal argument.
(v) Is the set of Pareto-optimal allocations of this economy non-empty? If not, justify your claim. If so, exhibit a Pareto-optimal allocation that cannot be sustained as a competitive equilibrium, specifying whether anything interesting can nevertheless about be said about such an allocation?

(vi) Is this economy an example, or a counter-example, to the claim that every core allocation can be sustained as a competitive equilibrium? Justify your answer.

(vii) Consider an alternative distribution of property rights in which each person owns a unit amount of the one commodity that he or she prefers to the other. Show the changes that are effected in the set of (a) Pareto optimal and (b) core allocations. Specify whether the set of competitive equilibria in this economy can be ranked on the basis of the amount of trade that is effected.
Part II

3. (20 minutes) (20 minutes) A strategic game \( \langle \{1, 2\}, (A_1, A_2), (u_1, u_2) \rangle \) is zero-sum if \( u_1(a) + u_2(a) = 0 \) for all \( a \in A_1 \times A_2 \).

Prove the following lemma.

If \( \langle \{1, 2\}, (A_1, A_2), (u_1, u_2) \rangle \) is a zero-sum strategic game, then

\[
\max_{y \in A_2} \min_{x \in A_1} u_2(x, y) = -\min_{y \in A_2} \max_{x \in A_1} u_1(x, y).
\]

Furthermore, \( y \in A_2 \) solves the problem \( \max_{y \in A_2} \min_{x \in A_1} u_2(x, y) \) if and only if it solves the problem \( \min_{y \in A_2} \max_{x \in A_1} u_1(x, y) \).

4. (30 minutes) Consider the following game between a principal and an agent who works for the principal. The agent's choice of effort \( e \in [0, 1] \) generates a monetary reward \( C \) to the principal. Specifically, \( C \) takes the value 1 with probability \( e \), and 0 with probability \( 1 - e \). Before the agent chooses his effort, the principal offers a wage contract \( (w_0, w_1) \in \mathbb{R}_+ \) where \( w_C \) is the wage paid when the reward observed is \( C \). The resulting utility to the principal is \( C - w_C \) and to the agent is \( P(w_C) - Q(e) \), where \( P \) and \( Q \) are differentiable and strictly increasing, \( P \) is strictly concave, \( Q \) is strictly convex, and \( P(0) = Q(0) = 0 \). Both agents maximize their expected utilities.

The game is played as follows. First the principal offers a wage contract to the agent. (This contract is accepted by the agent given the nonnegativity assumption above.) Then the agent chooses his effort.

(a) Define a pure strategy equilibrium for this game. (Ignore the acceptance decision of the contract since any feasible contract offered will be accepted).

(b) Show that if \( w_0 = w_1 \), then the agent does not have any incentive to work, i.e. his effort is zero.

(c) Show that if \( Q'(0) > P(1) \), then in equilibrium \( w_0^* = w_1^* = 0 \) and the agent chooses zero effort.

(d) Show that if \( Q'(0) \leq P(1) \), then in equilibrium \( 0 = w_0^* < w_1^* < 1 \) and the equilibrium effort of the agent is positive.

5. (30 minutes) Consider the following game of elections and media. There is an incumbent with two possible types \( \theta \in \{g, b\} \) with \( \Pr(\theta = g) = \gamma \), where \( g \) stands for "good" and \( b \) stands for "bad." Voters do not observe the type of the government, but learn about it through the media. If the incumbent is good,
the media observes no verifiable information. If the incumbent is bad, then with probability \( q \in [0, 1] \), it receives a verifiable signal to this effect. Only verifiable news can be printed. The incumbent can manipulate the news by offering a transfer \( t \) to the media. If the media accepts this offer, denoted by \( a = 1 \), then it gets a payoff of \( t/\tau \) where \( \tau \geq 1 \) is an exogenous transaction cost parameter. If the media rejects the offer, denoted by \( a = 0 \), then it receives a payoff of 1 when it has news, and 0 when it does not have news.

The voters re-elect the incumbent if there is no news, and elect a challenger if there is news (which can only be bad news given the assumptions above). Thus the payoff of the incumbent is \( \tau - ta \) if news is printed, and is \(-ta\) otherwise, where \( r \geq 0 \) is an exogenous parameter that captures the benefits of being re-elected.

The timing of the game is as follows:

1. The incumbent type \( \theta \) is realized.
2. The incumbent chooses a transfer \( t \geq 0 \).
3. The media decides whether to accept or reject \( t \).
4. The signal of the media is realized. If \( \theta = g \), the media observes no signal, denoted by \( s = \emptyset \). If \( \theta = b \), then the media observes \( s = b \) with probability \( q \) and \( s = \emptyset \) with probability \( 1 - q \).
5. If the media accepted the transfer, it reports \( s = \emptyset \) and receives \( t/\tau \). If it rejected the transfer, it reports the true signal and receives 1 if \( s = b \) and 0 if \( s = \emptyset \).

(a) Define a pure strategy equilibrium for this game between the incumbent and the media.
(b) Characterize an equilibrium.
Part III

6. (50 minutes) Consider an expected-profit maximizing monopoly insurer in a market with two type of risk-averse insurees; high-risk type whose probability of loss is $p^H$ and low-risk type whose probability of loss is $p^L$, $p^L < p^H$. The insurees’ type is private information. The proportion of the high-risk type in the population is $\lambda \in (0, 1)$. Assume that the insurees have identical risk attitudes, the same initial wealth, $w$, and face the possibility of a loss of $x$. The insurees’ “outside options” are their expected utilities without insurance.

(a) Formulate the monopoly problem.

(b) Which of the incentive compatibility and individual rationality constraints are binding and which are not?

(c) Describe in as much detail as you can the insurance policies that would be offered by the monopoly.

(d) How does the expected profit of the monopoly compare to the expected profit that could have been obtained if the information of individual risk type was public? Explain.

(e) Is the objective function of the monopoly implementable by a mechanism? Explain.

7. (30 minutes) For each of the following statements either prove it or give a counter-example:

(a) Let $X = (x_-, x_+)$ be an interval in the real line and denote by $\Delta(X)$ the set of simple probability distributions on $X$. Consider a preference relation $\succeq$ on $\Delta(X)$ that satisfies the axioms of expected utility theory. If $\succeq$ displays monotonicity with respect to first-order stochastic dominance and risk aversion then, for all $p \in \Delta(X)$ the certainty equivalent of $p$ exists and is unique.

(b) Consider a principal-agent problem with moral hazard. Suppose that both the principal and the agent are risk averse to the same degree, then the optimal incentive contract is first-best Pareto efficient.