MACROECONOMICS COMPREHENSIVE EXAM

August 2014

This exam consists of three Questions with equal weights.
**Question 1**

This exercise is about the impact of government debt on economic growth. It is motivated by the current debates on the long-run effects of the rise of government debt in advanced economies in the wake of the great recession. You will look at this question in the context of the AK model of endogenous growth.

The first question focuses on the consumption-saving problem of a single agent whose financial wealth is taxed by the government. The following questions embed this agent in a macroeconomic model of endogenous growth.

First, we consider the problem of an agent who maximizes

\[ \sum_{t=0}^{\infty} \beta^t \log c_t \]

subject to the budget constraint

\[ c_t + w_{t+1} = (R - \tau)w_t, \]

where \( w_{t+1} \) is the agent’s financial wealth at the end of period \( t \), and \( R - \tau \) is the gross return net of tax on financial wealth. Note that all the agent’s income comes from financial wealth—there is no labor income.

1.1) Using the first-order condition for this problem, show that

\[ c_t = (1 - \beta)(R - \tau)w_t. \]

Then show that:

(i) the agent’s financial wealth grows at a constant rate over time;
(ii) the rate of growth in financial wealth increases with the net return \( R - \tau \) and with the discount factor \( \beta \);
(iii) the rate of growth in financial wealth is positive if and only if \( \tau \) is smaller than \( R - 1/\beta \).

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Next, we consider an economy populated by a continuum of mass 1 of atomistic agents identical to the one described in the previous question. We denote macroeconomic quantities with upper-case letters, for example aggregate consumption is denoted by \( C_t \). Since population is normalized to 1, aggregate and per capita quantities are the same, \( c_t = C_t, \ w_t = V_t \) etc. but it will be conceptually useful to distinguish per capita and aggregate quantities.

We assume that financial wealth is composed of physical capital and government debt,

\[ W_t = K_t + D_t. \]
The economy's period-t GDP is equal to the capital's gross payoff net of depreciation,
\[ Y_t = (R - (1 - \delta)) K_t, \]
where \( \delta \) is the depreciation rate [take this statement as given, there is no need to prove or discuss]. You can think of the income on physical capital as the payment of income on financial securities (bonds, stocks) issued by firms. Note that output is proportional to the stock of capital, as in the AK model of endogenous growth.

The government's budget constraint is
\[ G_t + RD_t = D_{t+1} + \tau(K_t + D_t). \]
Government expenditures, \( G_t \), plus the principal and interest payment on government debt, \( RD_t \), are financed by the issuance of new debt, \( D_{t+1} \), plus the receipts of the tax on wealth, \( \tau(K_t + D_t) \).

1.2) Show that
\[ C_t + K_{t+1} + G_t = R K_t. \]

Explain how this equation can be interpreted as the "GDP identity" of introductory macro that relates GDP to private consumption, investment and government consumption.

1.3) For simplicity we assume from now on that government consumption is equal to zero,
\[ G_t = 0, \]
and we consider a balanced-growth path in which the stock of productive capital, \( K_t \), and the stock of government debt, \( D_t \), both grow at the same rate, denoted by \( \gamma \)
\[ \frac{K_{t+1}}{K_t} = \frac{D_{t+1}}{D_t} = 1 + \gamma. \]

Note that \( \gamma \) is also the GDP growth rate since GDP is proportional to productive capital. The initial stocks of capital and debt, \( K_1 \) and \( D_1 \) are part of the exogenous initial conditions.

Use the government budget constraint to find an expression giving the tax rate, \( \tau \), in terms of \( R \), \( \gamma \), and the share of government debt in total financial wealth, \( D_1/W_1 \). Does \( \tau \) increase or decrease with the growth rate \( \gamma \)? Give the economic intuition behind this result.

1.4) Using your answer to question 1, express the GDP growth rate \( \gamma \) in terms of \( \beta \), \( R \) and \( \tau \). Does the GDP growth rate increase or decrease with the tax rate, \( \tau \)? Give the economic intuition behind this result.

1.5) Combine the expression for \( \gamma \) that you derived in your answer to question 4 with the expression for \( \tau \) that you derived in your answer to question 3.
to express the growth rate $\gamma$ in terms of $\beta$, $R$ and $D_t/W_t$. (No other variable should appear on the right-hand side.)

Does the GDP growth rate increase or decrease with the share of government debt in total financial wealth? Give the economic intuition behind this result.

1.6) Following the Great Recession there have been debates between economists who think that it is important to implement fiscal consolidations early to stop the increase in government debt and economists who think that early fiscal consolidation is harmful. Which view do you think is more supported by this model? Explain why. What do you think is missing from the model to support the other view? [Give a heuristic answer—you are not asked to provide a full-fledged welfare analysis of a fiscal consolidation.]
Question 2

2.1) [30] Short Questions

Answer the following questions in a few paragraphs each including some algebra:

a) [10] What is the Friedman rule? Explain the intuition behind it. Why is the Friedman rule not useful for monetary policy in practice?

b) [10] What are the two main equilibrium conditions of the New Keynesian framework? Explain the intuition behind them. Do the two conditions fully describe the dynamics of a New Keynesian economy?

c) [10] What is a matching function in labor search models? Define its inputs and output. What properties does the matching function satisfy if there are “thick market effects” in the labor market? What if there are “crowding effects”? Explain the intuition.

2.2) [50] Wage Externalities and Bailouts

Assume an economy with two time periods $t = 0, 1$ in which there are two sets of private agents, households and financiers, as well as a government. Households are inactive in period 0. In period 1, they have an endowment $c$ and supply one unit of labor at the given market wage $w$. The government may levy a lump sum tax $T \in [0, c]$ on them that can be distributed to any of the agents. Households value consumption according to a standard utility function $u(c) = \ln c$.

a) What is the budget constraint of households?

b) What is the welfare of households as a function of the parameters given?

Financiers enter period 0 with one unit of good. They invest a fraction $\alpha$ of the good in a risky investment project that yields $A > 1$ per unit with probability $p < 1$ and 0 with probability $1-p$ where we assume $pA > 1$. They invest the remaining fraction $1 - \alpha$ in a safe project that yields a safe gross return of one. Both projects mature at the end of period 0. Denote the total payoff received by the financiers from their investments as $R$.

Due to a commitment problem, it is not possible for financiers to borrow from households. However, the government may decide to provide a bailout transfer $T$ from households to financiers. Financiers use their total net worth $k = R + T$ and invest it as capital; they also hire labor at the competitive market wage $w$ to engage in production using a technology $F(k, \ell) = k^\alpha \ell^{1-\alpha}$. They pay the wage bill and consume the remaining resources $\pi = F - w\ell$ which they value according to a linear utility function. In the following, we proceed by backward induction.

c) Express the profits of financiers $\pi$ as a function of $k$ and $w$. Then solve for the equilibrium $w$ and express $\pi$ as a function of $k$ only.
d) Express the welfare of workers as a function of \( R \), assuming that \( T = 0 \). What is \( \partial u / \partial k \)? Interpret. Can the effect you identified be described as a pecuniary externality?

e) If the government maximizes the welfare of workers only, then for what values of \( R \) does the government find it optimal to tax workers and provide a bailout transfer \( T > 0 \) to financiers in period 1? (Hint: what is the marginal cost of a bailout for workers? What is the marginal benefit for workers?)

f) Describe the optimal investment behavior of financiers in period 0 as a function of \( A \) and \( p \), assuming that \( T = 0 \).

g) If financiers expect the government to behave as you described in e) so that \( T \geq 3 \), then describe the optimal investment choices of financiers in period 0 as a function of \( A \) and \( p \). Interpret any differences from your answer in f).

h) If the government can credibly commit not to provide any bailouts, should it do so? How will this affect the investment choices of financiers?
Question 3

This question has two separate sections, 3.1 and 3.2.

Section 3.1:

Consider the following variation on the ISLM model for a closed economy:

\[ Y = C + I + G \]

\[ C = C(Y - T), \quad 0 < \gamma < 1 \]

\[ I = I(r), \quad \gamma^* < 0 \]

\[ (M/P)^d = L(i, C), \quad L_1 > 0, \quad L_2 > 0 \]

\[ (M/P)^d = M/P \]

\[ r = i - \pi^c \]

G, T, M, P, and \( \pi^c \) are exogenous.

A. How does the money demand function in this model differ from the money demand function in the textbook ISLM model? Why might the money demand function here be more realistic than the textbook specification?

B. Suppose that the government increases the level of taxes, T. What is the effect on output: positive, negative, or ambiguous? Prove your answer mathematically and explain the result by showing how the IS and LM curves shift. In this part of the question, assume the central bank holds the money supply constant.

C. How does the answer to Part B change if the central bank adjusts M to keep the interest rate constant when T rises? Here, prove your answer mathematically or explain the result by showing how the IS and LM curves shift (you do not need to do both).

D. How does the answer to Part B change if the nominal interest rate is close to zero before the increase in T? Prove your answer mathematically or explain the result by showing how the IS and LM curves shift. Here, assume as in Part B that the central bank holds the money supply constant.
Section 3.2

Consider the textbook Solow growth model. Let $s$ be the saving rate, $n$ the population growth rate, and $d$ the depreciation rate. Assume the aggregate production function is $Y = F(K, EL)$, where the $E$ is the efficiency of labor, which has a growth rate of $g$. Assume capital and labor are paid their marginal products.

In Parts A, B, and C, assume that the production function exhibits constant returns to scale, but do not assume a specific functional form for the production function.

A. In steady state, what are the growth rates of output; output per worker; the real wage; and the real interest rate (the MPK)? Explain your answers briefly (you do not need detailed proofs).

B. Suppose the population growth rate $n$ decreases. What is the effect on the steady state real interest rate: positive, negative, or ambiguous? Prove your answer mathematically or explain the result using graphs and economic reasoning.

C. Assume the economy is in steady state, and assume the following facts (which roughly fit the U.S. economy):

- The ratio of the capital stock to output is 2.5.

- Depreciation of capital is 10 percent of output.

- Capital income is 30 percent of output.

Derive numbers for the marginal product of capital and the depreciation rate. Show your calculations.

D. Assume that, initially, the facts in Part C hold and that $g=1\%$ and $n=2\%$. Also, assume that the aggregate production function is Cobb-Douglas. If $n$ falls from 2\% to 1\%, what happens to the steady-state MPK? Derive a numerical answer.