You have four hours for this exam. There are four parts. Parts I and II, which are long, each have a weight of 1/3 in your total score. Parts III and IV, which are shorter, each have a weight of 1/6.
Part I

There are debates about the implications of high government debt in advanced economies, in particular for inflation. This problem set looks at this issue using a simple continuous-time intertemporal optimizing model.

There is one homogeneous consumption good that is received as an endowment by consumers. The economy is populated by a mass 1 of identical atomistic consumers who maximize their intertemporal utility:

$$U_0 = \int_0^{+\infty} [c_t + v(m_t)] e^{-\rho t} dt,$$

where $c_t$ is the representative consumer’s consumption at time $t$, and $m_t = M_t/P_t$ is the consumer’s holdings of real money balances. Real money balances are equal to nominal money balances, $M_t$, divided by the nominal price level, $P_t$ (the price of the consumption good in terms of money). Real money balances yield utility to the consumer by helping him to implement transactions.

The budget constraints for the consumer and for the government are respectively given by:

$$P_t c_t + \dot{D}_t + \dot{M}_t = P_t y(1 - \tau) + i_t D_t,$n$$
$$\dot{D}_t + \dot{M}_t = (g - \tau y)P_t + i_t D_t,$$

where

- $D_t$ is the nominal level of government debt, which is held by the representative consumer
- $y$ is the real endowment of good received by the consumer (assumed to be constant)
- $\tau$ is the tax rate on the endowment (assumed to be constant)
- $i_t$ is the nominal interest rate
- $g$ is the level of real government expenditure (assumed to be constant)
- we use the notation $\dot{X} = dX/dt.$

The first equation says that the consumer spends his net endowment plus the return on government debt to buy consumption good and accumulate government debt and money. The second equation says that the government finances its primary deficit (first term on the r.h.s.) and the payment of the interest on its debt by issuing debt and money. The maturity of debt is equal to zero, implying that the whole stock of debt must be rolled over at each instant.

We will look throughout at perfect foresight equilibria with rational expectations.

1. We denote by lower-case variables the real counterparts of nominal variables, e.g., $d_t = D_t/P_t$. We denote by $a_t = d_t + m_t$ the total real assets of the
representative consumer, and by \( \pi_t = P_t / P_{t-1} \) the inflation rate. Show that the budget constraint of the consumer can be written,
\[
c_t + \hat{a}_t = (1 - \tau) y + \tau a_t,
\]
where \( r_t = i_t - \pi_t \) is the real interest rate and the symbol \( \hat{\ } \) stands for a term that you will derive in your computations. Interpret this term.

2. Show that the real interest rate \( r_t \) is equal to \( \rho \) by solving the consumer’s intertemporal optimization problem.

3. Derive the first-order condition for the consumer’s demand for real money balances by solving his optimization problem. Show that the demand for real money balances is a decreasing function of the nominal interest rate,
\[
m_t = m(i_t).
\]
Provide a mathematical derivation of this result as well as the underlying economic intuition.

4. Hereafter we assume that from period 0 onwards the economy is in a steady state with a constant level of consumption \( c_t \), a constant rate of inflation \( \pi_t \) and a constant level of debt \( \hat{d} \). Using the government budget constraint, find a relationship between \( d_t \), \( \tau \) and \( \pi_t \) that must hold in equilibrium.

5. Assume that (for political reasons for example) the tax rate \( \tau \) cannot be larger than an upper bound \( \overline{\tau} \). What is the maximum level of debt that the government can repay if it wants to maintain a zero inflation rate \( \pi = 0 \)? We denote this ceiling by \( \overline{d} \). Find a simple expression for \( \overline{d} \) involving \( \overline{\tau} y - \overline{g}, \rho \) and function \( m(\cdot) \). What is the maximum debt-to-GDP ratio, \( \overline{d}/y \), if \( \overline{\tau} - \overline{g}/y = 4\% \) and \( \rho = 2\% \)?

6. Assume that the level of debt is larger than the threshold determined in the previous question \( (d > \overline{d}) \). Can the government still pay its debt if inflation is now allowed to be nonzero? Should inflation be positive or negative?

7. Assume again that \( d > \overline{d} \). Explain why in order to maximize domestic welfare the tax rate should be set at its maximum level \( \overline{\tau} \). [To answer this question you will note that \( c = y - g \).] Given this, how does the required level of inflation vary with the level of real debt?

8. (Bonus question) The Friedman rule for monetary policy (proposed by economist Milton Friedman) is that the inflation rate should be equal to minus the real interest rate, \( \pi = -\rho \). Is the Friedman rule welfare-maximizing in this model? [To answer this question you may assume that there is a saturation level of money, \( m^* \), such that \( v'(m^*) = 0 \).] Is the Friedman rule a reason to maintain a low level of government debt?
Part II.

Short Discussion Question

Labor Supply and Consumption. This question asks you to explain some implications of models where labor supply and consumption are both chosen freely subject to a budget constraint. Define $z$ as the proportion of time spent in leisure activities (that is, not working) and $W_t, r$ and $\vartheta$ as the wage, interest, and time preference rates.

1. Assuming that the 'hat' operator is equivalent to a difference in logs (e.g., $\hat{\varepsilon}_{t+1} \equiv \log \varepsilon_{t+1} - \log \varepsilon_t$), explain the intuition behind the result that

$$\hat{z}_{t+1} \approx -\hat{W}_{t+1} + (r_{t+1} - \vartheta)$$

when preferences satisfy the "balanced growth" condition in a perfect foresight model and utility is logarithmic.

2. Discuss the relationship between this result and empirical evidence under two hypotheses about the reasons for employment fluctuations over the business cycle:
   a) They are driven mostly by temporary fluctuations in wages
   b) They are driven mostly by optimizing responses to interest rate shocks

3. Use the logic of the first order condition to discuss whether you would expect the addition of uncertainty about future consumption to change the implications of the frictionless model about the relationship between movements in consumption and equilibrium labor supply.
Long Discussion Question

In the period since the beginning of the Great Recession, a substantial literature has sprung up exploring the possibility that "uncertainty" may play an important role in business cycle dynamics.

This question asks you to use things you have learned in class to consider various aspects of this proposition.

1. **Consumption with Optimal Portfolio Choice.** CRRARateRisk shows that for a Merton (1969)-Samuelson (1969) consumer facing return \( \log R_{t+1} \sim \mathcal{N}(\mu - \sigma^2/2, \sigma^2) \) on the only financial asset available, the optimal marginal propensity to consume is approximately

\[
\kappa \approx \bar{r} - \rho^{-1} (\bar{r} - \varphi) - (\rho - 1) (\sigma^2/2) \tag{2}
\]

a) Use this equation to discuss the parametric restriction(s) under which an increase in unavoidable financial risk \( \sigma^2/2 \) will cause a (precautionary) decline in consumption (the risk is unavoidable because we have assumed the risky asset is the only financial asset available). Explain whether, under such parameter value(s), the income effect of an increase in \( \bar{r} \) outweighs the substitution effect, and explain in words the intuition for both the income effect and the substitution effect.

For the remainder of the question, assume that the consumer can choose how much to invest in the risky asset, so that financial risk can be avoided by reducing the share of the portfolio allocated to the risky asset. Portfolio-CRRAR shows the portfolio share \( \varsigma \) that an optimizing consumer will invest in a risky asset earning return \( \log R_{t+1} \sim \mathcal{N}(\mu - \sigma^2/2, \sigma^2) \) so that \( \log R = \bar{r} \) (where the subscriptless version of a variable denotes its expectation unless otherwise noted, e.g. \( R \equiv E_0[R_{t+1}] \)).

1 The remaining proportion \((1 - \varsigma)\) of the portfolio earns a riskless return \( r = \log R \), and we write the log expected return premium factor as \( \Phi \equiv R/R \) with log expected return premium \( \varphi \equiv \log \Phi = \bar{r} - r = \log R/R; \) optimal choice of \( \varsigma \) yields a portfolio whose realized return factor is written \( \tilde{R} \) where

\[
\log \tilde{R} \equiv \log E_0[\tilde{R}_{t+1}] = \varsigma \varphi + \bar{r}, \tag{3}
\]

and Portfolio-CRRAR shows that under these circumstances the optimal risky portfolio share is well approximated by

\[
\varsigma \approx \left( \frac{\varphi}{\rho \sigma^2} \right) \tag{4}
\]

\footnote{Note that the subscriptless version of the \( r \), is not equal to the mean: \( E_0[R_{t+1}] \approx \bar{r} - \sigma^2/2 \).}

\footnote{The bold font is used for the risky return and the narrow font for the safe return.}
while the variance of the return on the optimally-chosen risky portfolio is approximately
\begin{equation}
\sigma^2_x = \zeta^2 \sigma^2_x.
\end{equation}

b) Show that the precautionary effect of rate-of-return risk on the precautionary contribution to the MPC, after taking account of optimal portfolio adjustment, is
\begin{equation}
-(\rho - 1) \left( \frac{(\varphi/\rho)^2}{2\sigma^2_x} \right)
\end{equation}
and explain the intuition for the result that the size of the precautionary effect shrinks as the risk grows larger.

c) Now use the foregoing results to show that the MPC \( \kappa \) can be rewritten in terms of exogenous parameters (including the risk-free interest factor \( r \)) as
\begin{equation}
\kappa \approx r - \rho^{-1}(r - \vartheta) + (\rho - 1) \left( \frac{(\varphi/\rho)^2}{2\sigma^2_x} \right)
\end{equation}
and use this equation to explain why the total effect of an increase in risk on consumption is positive and explain why this result comes about.

d) Now show that the effect of an increase in risk on saving is given by
\begin{equation}
\varpi - \kappa = \rho^{-1}(r - \vartheta) + \left( \frac{\rho + 1}{2} \right) \left( \frac{\varphi^2}{\sigma^2_x \rho^2} \right)
\end{equation}
and explain how this result relates to the results obtained earlier. Comment, in particular, on the role played by our assumption that the consumer has no labor income (Hint: What further effect of interest rates is omitted when the consumer has no labor income?)

2. The next topic is the role played by labor income uncertainty. Use a buffer stock saving model to analyze the dynamics of consumption and saving that would result from a permanent increase in the perceived probability of unemployment in such a model. (Be sure to draw both the phase diagram and a diagram showing the time series dynamics of consumption and the saving rate).

3. Suppose the economy is composed of two classes: “Capitalists” (whose behavior is reasonably well captured by the Merton-Samuelson model) and “Workers” whose behavior is reasonably well captured by the buffer stock model. The figure below shows the dynamics of the U.S. personal saving rate over the 2007-2011 period, compared to its behavior in previous recessions. Under the assumption that the capitalist/worker distinction is a good way of summarizing different kinds of people's reactions, discuss whether this figure suggests that the behavior of “Capitalists” or of “Workers” was more important in explaining saving dynamics during the Great Recession.
4. Next we examine the relationship between uncertainty and labor supply. Use the intratemporal first order condition for the optimal allocation of time between labor and leisure to explain why a movement in consumption that is due to an increase in uncertainty should be accompanied by a movement of labor supply in the opposite direction. After explaining this, discuss how this implication relates to the actual movements of employment during the Great Recession. (Hint: This is not a trick question; employment did what you think it did, and so did consumption). If you had microeconomic data on the labor supply, income, and wealth of “workers” and “capitalists” how might you test the implications of the two models?

5. Now consider the effects of an increase in uncertainty on the behavior of a firm behaving according to the □ model of investment. Specifically, discuss two scenarios:

   a) **Small Open Economy.** In this scenario, the country we are considering is too small for the increase in uncertainty at the firm level (in this country) to have any effect on the global interest rate.

   b) **Global Financial Crisis.** In this scenario, the crisis is global and a consequence is that the riskfree interest rate drops.

6. Now discuss the effects of greater uncertainty on investment in the context of the model of capital market imperfections presented in class. Specifically, discuss the effects on the level and dynamics of investment and be sure to explain carefully the logic behind those effects.
7. Now use the Lucas Asset Pricing model to explain how the following facts about the Great Recession:
   
a) Interest rates on assets perceived to be safe (like U.S. Treasury bonds) fell sharply

b) Interest rate spreads (that is, the excess of a risky interest rate over the return on the safe asset) increased sharply

can be interpreted as the results of an increase in perceived uncertainty. Discuss also whether the model has anything useful to say about whether these two patterns could be understood according to each of the following alternative interpretations:

That movements in asset prices reflected
   
a) A decrease in expectations about long-run growth

b) An increase in the time preference factor $\beta$

c) An increase in the degree of risk aversion

8. Now produce a balanced assessment of the extent to which the important facts of the Great Recession can or cannot be explained using the models mentioned above as resulting from an increase in either (a) labor income uncertainty or (b) financial uncertainty or (c) both. If there are places where the models described above perform poorly, please highlight them and speculate about the kinds of models that might perform better.
References


PART III

Consider an individual who lives for two periods. The individual has no initial wealth, but earns labor income of \( Y_1 \) and \( Y_2 \) in the two periods. \( Y_1 \) is known, but \( Y_2 \) is random, with \( E[Y_2] = Y_1 \). The individual pays taxes of \( t_1 Y_1 \) in the first period and \( t_2 Y_2 \) in the second period, where \( t_1 \) and \( t_2 \) are tax rates. The individual can borrow and lend at an interest rate of zero. Thus second-period consumption is \( C_2 = (1-t_1)Y_1-C_1+(1-t_2)Y_2 \). The individual chooses \( C_1 \) to maximize expected lifetime utility, \( U(C_1)+E[U(C_2)] \), where \( U(C_i) \) is utility in period \( i \).

Throughout this question, consider two specific utility functions:

1. \( U(C_1) = aC_1 - bC_1^2 \), where \( a \) and \( b \) are positive constants
2. \( U(C_1) = \ln(C_1) \), where \( \ln \) means natural logarithm

A. Which of the two utility functions is more realistic? Briefly explain.

B. For each of the two utility functions, say whether the consumer chooses \( C_1 > E[C_2] \), \( C_1 = E[C_2] \), or \( C_1 < E[C_2] \). Prove your answers mathematically and give brief economic explanations.

C. Suppose the government lowers \( t_1 \) and raises \( t_2 \) by the same amount, so that its total expected revenue, \( t_1 Y_1 + t_2 E[Y_2] \), is unchanged. For each of the two utility functions, does this tax change raise \( C_1 \), lower \( C_1 \), or leave \( C_1 \) unchanged? Prove your answers mathematically and give brief economic explanations.

D. Briefly discuss the relevance of your results to the debate over Ricardian Equivalence.
PART IV

Consider each of the following changes in an economy. In each case, assume the change is permanent. Say how the following macroeconomic variables are affected: the level and growth rate of output, nominal and real interest rates, the real wage, and inflation. Describe what happens to these variables in the short run and in the long run. Base your answers on standard macroeconomic models and say what models you are using. (You do not need to prove your answers; you can just say that a certain model implies certain effects.)

A. A decrease in government spending.

B. An increase in the growth rate of the money supply.

C. An increase in the growth rate of total factor productivity.

D. An increase in the retirement age for workers.