An Inventory of Simple Monetary Policy Rules in a New Keynesian Macroeconomic Model*

Thomas A. Lubik†
Department of Economics
The Johns Hopkins University

Massimiliano Marzo‡
Department of Economics
University of Bologna

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Abstract

We derive necessary and sufficient conditions for simple monetary policy rules that guarantee equilibrium determinacy in the New Keynesian monetary model. Our modeling framework is derived from a fully specified optimization model that is still amenable to analytical characterisation. The monetary rules analyzed are variants of the basic Taylor rules ranging from simple inflation targeting (current, forward, backward), to the canonical Taylor rules with and without inertial nominal interest rate patterns. We establish that determinacy obtains for a wide range of policy parameters, especially when the monetary authority targets output and smoothes interest rates. Contrary to other results in the literature we do not find a case for super-inertial interest rate policy

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1 Introduction

This paper derives parameter restrictions for simple monetary policy rules which deliver a fully determinate equilibrium in an otherwise standard monetary general equilibrium model. This model has become the workhorse in the literature on monetary policy with the seminal contribution of Rotemberg and Woodford (1997). The main result from our analysis is that the monetary authority should implement an aggressive anti-inflationary stance irrespective of whether the interest-rate rule is current-, forward- or backward-looking, or whether targeting the output gap is an issue or not. However, a rule with a large coefficient on inflation targeting does not necessarily lead to determinacy when there is no interest rate inertia

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[†] Mergenthaler Hall, 3400 N. Charles St, Baltimore, MD 21230. Email: thomas.lubik@jhu.edu. Tel: +1 410 516 5564. Fax: +1 410 516 7600.

[‡] Corresponding author. 2, Piazza Scaravilli - 40126 Bologna BO, Italy. Tel.: ++39-051-209-8019; Fax: ++39-051-209-8040; Email: marzo@economia.unibo.it

or output targeting. When these are introduced, determinacy obtains for a wide range of parameter constellations, thus 'supporting' the monetary authority's inflation response. Notably, and contrary to policy recommendation for super-inertial interest-rate setting, an inertial coefficient lower than one is found to be a necessary determinacy condition.

This paper presents a taxonomy of determinacy for a simple forward-looking monetary model. Many of the results have appeared in a variety of papers[1], but, to the best of our knowledge, not in a unified framework. The rules analyzed here include pure inflation targeting (current, forward and backward looking); rules with both inflation and output targeting (current, forward and backward looking), and rules with interest rate inertia (current and forward looking). As discussed by McCallum and Nelson (1999), the rules employed in this paper are all 'operational' in the sense that policy makers either react to (i) lagged values of inflation and output deviations from their respective targets; (ii) or react to their expectations of current values of inflation deviations and the output gap. A key element of our approach is the formal derivation of the aggregate supply equation under a different specification for modeling nominal rigidities, namely quadratic costs of price adjustment *à la* Rotemberg (1982). Moreover, we show how the boundaries of the determinacy region explicitly depend on the structural model parameters.

Recent literature explores the characteristics of monetary dynamic general equilibrium models under both empirical and theoretical aspects. Papers such as King and Watson (1996) or Kim (2000) analyze the properties of this modelling framework with respect to its ability to replicate the basic business cycle regularities. The contribution of Rotemberg and Woodford (1997) was to shift the emphasis of this literature towards the performance of such models when a Taylor (1993) type monetary is applied. The conference volume edited by Taylor (1999) presents a wide collection of papers in this vein. The issue of determinacy, however, has come to the forefront of this literature only recently. It has been recognized that the application of monetary policy rules can be destabilizing. An interest rate policy that is not aggressive enough in the face of rising inflation can lead to adverse outcomes where non-fundamental or 'sunspot' shocks can affect aggregate dynamics which would not be present otherwise. Equilibrium determinacy thus becomes a matter of policy design.

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[1] These include Bullard and Mitra (2002), Carlstrom and Fuerst (2000, 2001), Clarida et al. (2000), Woodford (2003).

This line of analysis has been succinctly summarized by Woodford (2003) who also presents some results in a model similar to ours. Bullard and Mitra (2002) address determinacy in a set of models within the framework of least-squares learning. They do not, however, derive their model from first principles, which could violate the cross-equation restrictions embedded in our reduced form. Benhabib et al. (2001) shadow most of our results, albeit in a continuous time setting. Carlstrom and Fuerst (2000) are closest to this paper, but their use of a different timing convention for money holdings leads to different determinacy results. None of these papers, however, examines as wide a taxonomy of rules as that proposed in the current work.

The paper is organized as follows. In the next section we present a simple monetary model usable for policy analysis, the reduced-form of which is derived from a fully optimization based model of consumer and firm choice. In Section 3, we derive determinacy conditions for pure inflation targeting in several variants, while Section 4 adds output targeting. Section 5 studies the role of interest rate inertia in the policy rule. Section 6 summarizes and concludes.

2 A Canonical Model for Monetary Policy Analysis

2.1 A Representative Household

The economy is populated by a representative household that derives utility from consumption c and real balances $\frac{M}{P}$, and disutility from working, where l denotes the labor supply:

$$U = \sum_{t=0}^{\infty} \beta^t \left[\frac{c_t^{1-\frac{1}{\sigma}} - 1}{1 - \frac{1}{\sigma}} + \frac{\chi}{1-\varepsilon} \left(\frac{M_t}{P_t} \right)^{1-\varepsilon} - \frac{1}{1+\frac{1}{\eta}} l_t^{1+\frac{1}{\eta}} \right]. \tag{1}$$

P is the economy-wide nominal price level which the household takes as given. σ ($0 < \sigma < \infty$) denotes the intertemporal substitution elasticity, the inverse of which is the coefficient of relative risk aversion. For $\sigma = 1$ consumption enters period utility in logarithmic form $\log c_t$. $\chi > 0$ is a scaling parameter, while ε^{-1} is the partial interest elasticity of money demand. η ($0 < \eta < \infty$) is the elasticity of labor supply. For $\eta \to 0$ labor is fixed, whereas labor becomes perfectly elastic for $\eta \to \infty$. $0 < \beta < 1$ is the constant subjective discount factor.

The household supplies labor services to the firm sector for which it receives the real wage w. It has access to one-period nominal government bonds B that pay (gross) interest R. Furthermore, as the owner of the firms technology it receives aggregate residual profits Ω and has to pay real lump-sum taxes τ to the government. Consequently, the household maximizes (1) subject to the budget constraint:

$$c_t + \frac{B_t}{P_t} + \frac{M_t}{P_t} - \tau_t = w_t l_t + \frac{M_{t-1}}{P_t} + R_{t-1}\frac{B_{t-1}}{P_t} + \Omega_t \tag{2}$$

and the usual transversality condition on asset accumulation that rules out Ponzi-schemes. Initial conditions are given by $M_0 > 0, B_0 > 0$.

The household's first-order conditions are then given by:

$$\begin{aligned} c_{t+1} &= \left(\beta \frac{R_t}{\pi_{t+1}}\right)^{\sigma} c_t, && (3)\\ \frac{M_t}{P_t} &= \chi^{\frac{1}{\varepsilon}} \left(\frac{R_t}{R_t - 1}\right)^{\frac{1}{\varepsilon}} c_t^{\frac{1}{\varepsilon\sigma}}, && (4)\\ l_t &= c_t^{-\frac{\eta}{\sigma}} w_t^{\eta}. && (5) \end{aligned}$$

Equation (3) is the usual Euler-equation. Consumption growth is an increasing function of the real interest rate, which is defined as the nominal interest rate adjusted for expected (gross) inflation $\pi_{t+1} = \frac{P_{t+1}}{P_t}$. Equation (4) represents a standard, interest-sensitive money-demand schedule. Given R_t and consumption c_t this equation fully determines real money balances. Finally, labor supply (5) is an increasing function of the real wage adjusted for marginal utility.

2.2 Monopolistically Competitive Firms

The production sector is described by a continuum of monopolistically competitive firms equally distributed on the unit interval $[0, 1]$. Each individual firm $j \in (0, 1)$ faces a downward-sloping demand curve for its own differentiated product $y_t(j)$:

$$P_t(j) = \left(\frac{y_t(j)}{y_t}\right)^{-1/\theta} P_t. \tag{6}$$

Such a demand function can be derived in the usual way from Dixit-Stiglitz (1977) preferences, where $P_t(j)$ is the profit-maximizing price consistent with production level $y_t(j)$. The parameter θ ($1 < \theta < \infty$) is the elasticity of substitution between two differentiated goods. As $\theta \to \infty$ the demand function becomes perfectly elastic, and the differentiated goods effectively become substitutes. The aggregate price level P_t and aggregate demand/output y_t are seen as beyond the control of the individual firm. The assumption of monopolistic competition allows us to rationalize the firms' price-setting behavior. However, this is not enough by itself to generate real effects of monetary shocks[2]. Additionally, nominal variables, such as profit-maximizing prices, have to respond sluggishly.

Nominal price stickiness is introduced via the assumption of quadratic adjustment costs:

$$AC_t(j) = \frac{\varphi}{2}\left(\frac{P_t(j)}{P_{t-1}(j)} - \pi\right)^2 y_t \tag{7}$$

When an individual firm wants to change its price above and beyond the general trend in prices, given by the inflation rate π, it incurs 'menu costs' in the form of lost output. The parameter $\varphi \geq 0$ governs the degree of stickiness in the economy. Assuming Cobb-Douglas production $y_t(j) = l_t^{\alpha}(j)$ the firm j chooses factor inputs $l_t(j)$ to maximize (real) profits:

$$\Pi_t\left(j\right) = \sum_{t=0}^{\infty} \rho_t \Omega_t(j) = \sum_{t=0}^{\infty} \rho_t \left[\frac{P_t(j)}{P_t} y_t(j) - w_t l_t(j) - AC_t(j)\right], \tag{8}$$

subject to (6) and the production function. Note that ρ_t is the time-dependent discount factor that all firms use to evaluate future profit streams. Under the assumption of perfect insurance markets $\rho_t = \beta\left(\frac{c_t}{c_{t+1}}\right)^{\frac{1}{\sigma}}$. In other words, since the household is the recipient of the firms' residual payments it 'directs' firms to make decisions based on its intertemporal marginal rate of substitution.

All firms are heterogeneous *ex ante*, so that each producer makes decisions based on his own demand curve. In order for the model to remain tractable we make the following assumption: Firms behave in an identical way so that individual firms can be aggregated into a single representative, monopolistically competitive firm; they are thus homogeneous *ex post*. The firm's first-order condition is (after imposing homogeneity):

$$w_t = \alpha \frac{y_t}{l_t}\left(1 - \frac{1}{\varepsilon_t}\right), \tag{9}$$

where

$$\frac{1}{\varepsilon_t} = \frac{1}{\theta}\left[1 - \varphi\left(\pi_t - \pi\right) + \beta\left(\frac{c_t}{c_{t+1}}\right)^{\frac{1}{\sigma}} \varphi\left(\pi_{t+1} - \pi\right)\pi_{t+1}^2 \frac{y_{t+1}}{y_t}\right] \tag{10}$$

ε_t can be interpreted as the output demand elasticity augmented by the utility-weighted cost of price adjustment. Note that the mark-up of price over marginal cost is $\mu_t = \left(1 - \frac{1}{\varepsilon_t}\right)^{-1}$,

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[2] Aside from any expected inflation effects working through the Fischer equation.

so that the steady state (gross) mark-up is $\frac{\theta}{\theta-1} > 1$. The substitution elasticity thus provides an index of the degree of monopolistic distortion in the economy. In the perfectly competitive case, with $\theta \to \infty$, the mark-up is unity. Furthermore, when prices are perfectly flexible ($\varphi = 0$), the mark-up is constant. With sticky prices on the other hand, the mark-up becomes endogenous and functions as a transmission channel for both real and nominal shocks (see eq. (9)). An increase in ε_t reduces the mark-up and is expansionary due to the reduction of the monopolistic distortion.

2.3 Equilibrium and Local Dynamics

The dynamic equilibrium of the economy is described by the first-order conditions of the representative household, (2), (3), (5), and the representative monopolistically competitive firm, (9), (10). In order to close the model we need to specify the behavior of the government. We assume that the fiscal authority rebates any inflation tax revenue to the household:

$$\tau_t = \frac{M_t - M_{t-1}}{P_t} + \frac{B_t - R_{t-1}B_{t-1}}{P_t}, \tag{11}$$

which allows us to abstract from government debt dynamics.[3] Substituting the government budget constraint (11) into (2) and using (7), (8), we find that the social resource constraint is: $c_t = y_t \left[1 - \frac{\varphi}{2}(\pi_t - \pi)^2\right]$. The specification of monetary policy will be discussed in the next section.

We now proceed to log-linearize the equations describing an equilibrium around the deterministic steady state. Denoting $\widetilde{x}_t \equiv \log x_t - \log x$ as the (approximate) percentage deviation of x_t from its steady state level x, the Euler-equation (3) can be written:

$$\widetilde{c}_{t+1} = \sigma\widetilde{R}_t - \sigma\widetilde{\pi}_{t+1} + \widetilde{c}_t. \tag{12}$$

Since $\widetilde{c}_t = \widetilde{y}_t$, this equation describes the time path of aggregate output via the consumption-saving decision of the household. It represents the demand-side of the economy and is often interpreted as an expectations-augmented 'IS'-curve. The log-linear labor-supply equation is simply:

$$\widetilde{l}_t = \eta\widetilde{w}_t - \frac{\eta}{\sigma}\widetilde{c}_t, \tag{13}$$

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[3] In Leeper's (1991) terminology, the fiscal authority follows a 'passive' policy rule in that it prevents the time path of debt from becoming explosive by aggressively changing taxes. Lubik (2003) studies monetary policy rules and indeterminacy for a wider range of fiscal behavior.

while (9) can be approximated as:

$$\widetilde{w}_t = \widetilde{y}_t - \widetilde{l}_t + \frac{1}{\theta - 1}\widetilde{\varepsilon}_t. \tag{14}$$

Finally, the output-demand elasticity, as defined in (10), is:

$$\widetilde{\varepsilon}_t = \varphi\widetilde{\pi}_t - \beta\varphi\widetilde{\pi}_{t+1}. \tag{15}$$

We now use $\widetilde{c}_t = \widetilde{y}_t$, and $\widetilde{y}_t = \alpha\widetilde{l}_t$, in equations (13) and (14), and after a few simple steps we find:

$$\widetilde{\varepsilon}_t = \frac{\theta - 1}{\alpha}\left(\frac{1}{\eta} + \frac{\alpha}{\sigma} + 1 - \alpha\right)\widetilde{y}_t. \tag{16}$$

Note that from the definition of the mark-up $\widetilde{\mu}_t = -\frac{1}{\theta-1}\widetilde{\varepsilon}_t$ so that:

$$\widetilde{\mu}_t = -\frac{1}{\alpha}\left(\frac{1}{\eta} + \frac{\alpha}{\sigma} + 1 - \alpha\right)\widetilde{y}_t. \tag{17}$$

An increase in the mark-up is thus accompanied by a decline in production. Furthermore, from equation (15), we find that current inflation is expansionary by lowering the mark-up.

We can now combine (15) and (16) to derive a dynamic inflation equation:

$$\beta\widetilde{\pi}_{t+1} = \widetilde{\pi}_t - \kappa\widetilde{Y}_t, \tag{18}$$

where

$$\kappa = \frac{\theta - 1}{\varphi}\left(\frac{1}{\alpha\eta} + \frac{1}{\sigma} + \frac{1-\alpha}{\alpha}\right) > 0.$$

Equation (19) is often referred to as an intertemporal 'AS'-equation or Phillips-curve that determines the evolution of the economy's output as a function of current and expected inflation rates. κ^{-1} measures the elasticity of aggregate supply with respect to inflation. In the benchmark case of log-utility, linear leisure and linear production this coefficient reduces to $\kappa = \frac{\theta-1}{\varphi}$. κ^{-1}, which is increasing in the stickiness parameter φ and decreasing in θ, can thus be interpreted as an index of the overall degree of distortion in the economy. The more distorted the economy, the more responsive is output to inflation, both current and expected. While κ is a sufficient statistic for the slope of the Phillips-curve and the focus of many empirical studies, it is actually a non-linear function of underlying structural parameters, and thus embodies the cross-equation restrictions inherent in a fully specified general equilibrium model. Further note that output is more volatile to inflation the more elastic the labor supply is, the lower the curvature of the production function and the less risk averse households are.

The reduced form of our optimization-based structural model is then as follows:

$$\begin{aligned} \beta\widetilde{\pi}_{t+1} &= \widetilde{\pi}_t - \kappa\widetilde{Y}_t, && (19) \\ \widetilde{Y}_{t+1} + \sigma\widetilde{\pi}_{t+1} &= \sigma\widetilde{R}_t + \widetilde{Y}_t. && (20) \end{aligned}$$

In order to close this equation system we now have to specify a time path for the nominal interest rate $\widetilde{R}_t$. We do so by assuming that the monetary authority follows an interest rate rule that maps deviations of the inflation rate and output from their respective target levels into the interest rate instrument. In the next section we study simple inflation rules, while in Section 4 we analyze variants of the classic Taylor (1993)-rule. Section 5 then discusses the case of interest-rate inertia.

3 Inflation Targeting and Indeterminacy

The central bank is assumed to follow an interest-rate rule of the form:

$$R_t = R\left(\frac{\pi_{t+i}}{\pi}\right)^{\phi_\pi}, \tag{21}$$

where we only consider rules for which $\phi_\pi \geq 0$. Following Leeper (1991) we refer to rules for which $\phi_\pi > 1$ as active, and passive otherwise. The timing-index i takes on values $-1, 0, 1$, which correspond to backward-looking, current, and forward-looking inflation targeting, respectively. R and π are the steady state levels of the corresponding variables. Writing this equation in log-linear form:

$$\widetilde{R}_t = \phi_\pi\widetilde{\pi}_{t+i} \tag{22}$$

allows us to substitute out the nominal interest rate from (19) - (20). This yields a first-order difference equation system in π_t and Y_t only, which, depending on the timing assumption in the inflation rule, is two- or three-dimensional and can be analyzed analytically.

In what follows, we will use the terms 'determinacy', 'uniqueness', and their variants interchangeably. We also focus on the deterministic case for expository reasons since the determinacy properties of a stochastic version of the model are identical. Checking for determinacy involves computing the auto-regressive matrix of the equation system and evaluating for which parameter constellations its roots are inside or outside the unit circle. Since analytical computation of the roots themselves is almost always impossible, we use the Schur-Cohn criterion on the localization of the roots of a polynomial instead. An Appendix contains general conditions for the 2x2 and 3x3 case.

3.1 Current-Looking Rule

We first study the determinacy properties of a current inflation-targeting rule $\widetilde{R}_t = \phi_\pi \widetilde{\pi}_t$. This case has been previously analyzed by Carlstrom and Fuerst (2000) amongst others. We substitute the policy rule into (19) - (20). This leads to the reduced-form system in inflation and output:

$$\begin{bmatrix} \widetilde{\pi}_t \\ \widetilde{Y}_t \end{bmatrix} = \begin{bmatrix} \beta^{-1} & -\frac{\kappa}{\beta} \\ \sigma\left(\phi_\pi - \beta^{-1}\right) & 1 + \frac{\kappa\sigma}{\beta} \end{bmatrix} \begin{bmatrix} \widetilde{\pi}_{t-1} \\ \widetilde{Y}_{t-1} \end{bmatrix}. \tag{23}$$

We establish determinacy properties in the following Theorem.

Theorem 1 *Under a current inflation-targeting rule equilibrium determinacy obtains if and only if monetary policy is active, that is, $\phi_\pi > 1$.*

Proof. Since $\widetilde{\pi}$ and $\widetilde{Y}$ are jump variables, both eigenvalues have to be outside the unit circle for determinacy. The determinant $det = \frac{1}{\beta}\left(1 + \phi_\pi \kappa\sigma\right) > 1$ for $\phi_\pi \geq 0$. We thus need to check whether $1 + tr(A) + \det(A) > 0$. The trace of the coefficient matrix is $tr = 1 + \frac{1}{\beta}\left(1 + \kappa\sigma\right)$, which is strictly positive, so that this condition always holds. Secondly, we require $1 - tr(A) + \det(A) > 0$. Or: $1 + \frac{1}{\beta}\left(1 + \kappa\sigma\right) < 1 + \frac{1}{\beta}\left(1 + \phi_\pi \kappa\sigma\right)$. This inequality holds if and only if $\psi > 1$. ■

The policy prescription for the monetary authority is simple: raise the nominal rate more than proportionally than movements in inflation. Note that the determinacy region does not depend on structural parameters of the model. It is this result that has become the hallmark of the discussion about the preferability of inflation-targeting since the central bank does not need to know the structure of the economy. Since this two-equation model contains two jump-variables the only other possible outcome is an indeterminate equilibrium where one root of the system is inside the unit circle.

We can develop some economic intuition for this result by trying to construct a sunspot equilibrium. Suppose that the realization of a sunspot variable leads agents to believe that the current inflation rate is above its steady state value, $\pi_0 > \pi$, and adjustment dynamics are monotone thereafter, $\pi_{t+1} < \pi_t, t = 0, 1, \ldots$. The monetary authority responds by raising the nominal interest rate, which, under an active policy, increases the real rate. From the Euler-equation (3) this implies positive consumption growth since agents increase current savings. Although the inflation rate is falling towards the steady state, active policy keeps the real rate from declining so that consumption growth remains positive in future periods. Note that the inflation path implies that output increases (see eq. (16)), which is inconsistent with current consumption declining to support an increase in savings. $c_0 > c$, however, and subsequent positive consumption growth is obviously not an equilibrium path with steady state adjustment dynamics. A strictly active policy thus rules out self-fulfilling expectations.

Suppose that $\phi_\pi < 1$. The (expected) real rate declines, consumption growth is negative. Current consumption increases, which leads firms to raise prices in combination with lowering the mark-up, and production is stimulated. From (16), this is consistent with positive current inflation, which validates the initial assumption of sunspot-driven inflation expectations. As inflation falls towards its steady state, passive policy keeps the real rate low, and output and consumption return to the steady state at a slower and slower rate.

3.2 Forward-Looking Rule

The monetary authority is now assumed to use the rule $\widetilde{R}_t = \phi_\pi \widetilde{\pi}_{t+1}$. Such a rule is often rationalized by arguing that the central bank has a distinct information advantage over the private sector in that it has earlier and more detailed access to economic data as they become available. This allows it to preemptively react to any inflationary signals which the private sector may not be fully aware of. Many empirical specifications of monetary rules use in fact a forward-looking inflation component. Our findings on the determinacy properties of this rule are summarized in the following Theorem.

Theorem 2 *Under a forward-looking inflation-targeting rule equilibrium determinacy obtains if and only if* $1 < \phi_\pi < 1 + 2\frac{1+\beta}{\kappa\sigma}$.

Proof. The matrix of the system is given by

$$\begin{bmatrix} 1/\beta & -\kappa/\beta \\ \frac{\sigma}{\beta}(\phi_\pi - 1) & 1 - \frac{\kappa\sigma}{\beta}(\phi_\pi - 1) \end{bmatrix},$$

with trace $tr = 1 + \frac{1}{\beta} - \frac{\kappa\sigma}{\beta}(\phi_\pi - 1)$ and determinant $\det = \frac{1}{\beta}$. A unique equilibrium requires two explosive roots. Since $\det > 1$ always, we need to evaluate whether $-(1 + \det) < tr < 1 + \det$. The right-hand side of this relation imposes the restriction $\phi_\pi > 1$, while the left-hand side requires $\phi_\pi < 1 + 2\frac{1+\beta}{\kappa\sigma}$. The Theorem follows immediately. ■

A necessary condition for uniqueness is that monetary policy is active, but this is not sufficient. With a forward-looking rule there is an upper bound on the inflation coefficient. How tight is this bound? Note that the determinacy region disappears as either prices become perfectly flexible ($\varphi = 0$) or goods become perfect substitutes ($\theta \to \infty$). In other words, the upper bound is increasing in the distortion-index κ^{-1}. Thus, the fact that an economy is distorted enough guarantees existence and uniqueness for an active forward-looking rule. Why is the determinacy region constrained from above? Recall that the possibility of sunspot equilibria hinges on whether the expected real rate (and therefore expected consumption growth) declines in response to monetary policy actions. In the previous case we could rule out indeterminacy by arguing that active policy implies sunspot dynamics that are explosive and, thus, do not constitute an equilibrium. Under forward-looking policy the expected real rate turns out to be $\sigma(\phi_\pi - 1)\widetilde{\pi}_{t+1}$ which is negative if $\widetilde{\pi}_{t+1} < 0$.

We now consider sunspot inflation dynamics $\widetilde{\pi}_t > 0 > \widetilde{\pi}_{t+1}$.[4] Since firms belief that inflation is positive they lower their mark-ups and production expands. This effect is amplified if expectations are such that future inflation will be below its steady state value. The central bank reacts to these deflation expectations by aggressively cutting the nominal rate, which stimulates consumer spending due to a decline in desires savings. The increase in demand prompts firms to raise prices which confirms the initial expectations of higher inflation. The subsequent decline in demand then coincides with firms cutting prices. The sunspot inflation path is not monotone in this case. Although policy is active, the central bank cannot prevent equilibrium indeterminacy since it is overly aggressive in lowering rates in the face of falling future inflation. When combining (19) with (20) we have that $\Delta\widetilde{y}_{t+1} = -\frac{\kappa\sigma}{\beta}(\phi_\pi - 1)\widetilde{y}_t + \sigma(\phi_\pi - 1)\widetilde{\pi}_t$. For a given ϕ_π, indeterminacy ($\Delta\widetilde{y}_{t+1} < 0$) becomes more of a possibility the bigger κ is, i.e. the less distorted the economy is. For expected inflation coefficients below the lower bound $\phi_\pi < 1$, the same reasoning as in the

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[4] Monotone dynamics can be ruled out since it immediately implies an explosive output path.

previous section can be applied.

3.3 Backward-Looking Rule

We conclude this section by studying the rule $\widetilde{R}_t = \phi_\pi \widetilde{\pi}_{t-1}$. The motivation for adopting such a rule lies in the intervention and observation lag usually related with monetary policy. The determinacy result is contained in the following Theorem.

Theorem 3 *Under backward-looking inflation targeting equilibrium determinacy obtains if and only if* $1 < \phi_\pi < 1 + 2\frac{1+\beta}{\kappa\sigma}$.

Proof. The equation system contains two jump and one predetermined variable. Determinacy therefore requires two explosive and one stable root. General conditions for which this holds can be found in the Appendix. The system matrix is

$$\begin{bmatrix} 1/\beta & -\kappa/\beta & 0 \\ -\sigma/\beta & 1+\frac{\kappa\sigma}{\beta} & \sigma \\ \phi_\pi & 0 & 0 \end{bmatrix},$$

with the coefficients of the characteristic equation $\lambda^3 + A_2\lambda^2 + A_1\lambda + A_0$ given by $A_2 = -\left(1 + \frac{1}{\beta} + \frac{\kappa\sigma}{\beta}\right)$, $A_1 = \frac{1}{\beta}$, $A_0 = \frac{\kappa\sigma}{\beta}\phi_\pi$. Under Case I Condition (A.1) requires $\frac{\kappa\sigma}{\beta}(\phi_\pi - 1) < 1$, while (A.2) leads to $\phi_\pi > 1 + 2\frac{1+\beta}{\kappa\sigma}$. These restrictions are mutually exclusive. However, (A.3) and (A.4) are their respective alternatives so that $1 < \phi_\pi < 1 + 2\frac{1+\beta}{\kappa\sigma}$. Under Case II (A.5) has to hold, that is, $\left(\frac{\kappa\sigma}{\beta}\phi_\pi\right)^2 + \frac{\kappa\sigma}{\beta}\phi_\pi\left(1 + \frac{1}{\beta} + \frac{\kappa\sigma}{\beta}\right) + \frac{1}{\beta} - 1 > 0$. This is always true for the entire admissible parameter space, by which Case III can be ruled out. The Theorem follows immediately. ■

Remarkably, backward- and forward-looking rules both require the same policy parameter restrictions for determinacy. This implies a 'symmetry' - at least as far as determinacy is concerned -, of forward- and backward-looking behavior of the central bank. Monetary policy has to be active in raising the real rate, but not too active.

4 Taylor-Rules and Indeterminacy

We now investigate the determinacy properties of the economy when the central bank follows a Taylor-Rule:

$$R_t = R\left(\frac{\pi_{t+i}}{\pi}\right)^{\phi_\pi}\left(\frac{y_{t+j}}{y}\right)^{\phi_y}, \tag{24}$$

with ϕ_π, $\phi_y \geq 0$, and y is the steady state level of output. As before let $i, j \in \{-1, 0, 1\}$, but we are also interested in permutations of the timing decision. In log-linear form the rule is:

$$\widetilde{R}_t = \phi_\pi \widetilde{\pi}_{t+i} + \phi_y \widetilde{y}_{t+i}. \tag{25}$$

4.1 Current-Looking Taylor-Rule

With a current-looking rule $\widetilde{R}_t = \phi_\pi \widetilde{\pi}_t + \phi_y \widetilde{y}_t$, parameter restrictions for equilibrium determinacy are provided by the following theorem.

Theorem 4 *Under a current-looking Taylor-rule equilibrium determinacy obtains if and only if* $\max\left\{1 - \frac{1-\beta}{\kappa}\phi_y, 0\right\} < \phi_\pi$.

Proof. See Appendix. ■

Including output targeting dramatically increases the options available to the monetary authority. Even a small output coefficient allows the central bank to react only weakly to inflation. The reason for this result is, of course, that $\widetilde{\pi}_t$ and $\widetilde{y}_t$ are connected via the aggregate supply equation (19). All else being equal, the central bank could raise the real rate by responding to output movements alone. This is compounded by a feedback effect from the Phillips-curve which determines the rate at which output deviations are translated into inflation dynamics. The rate at which an isolated response to inflation is traded off against an (isolated) response to output is given by $\frac{1-\beta}{\kappa}$, the distortion index adjusted for the rate at which inflation decays. Again, the more distorted the economy the more powerful is output targeting.

4.2 Forward-Looking Taylor Rules

In their empirical study of actual monetary policy behavior Clarida, Gali, and Gertler (2000), henceforth CGG, found that a Taylor-Rule with forward-looking inflation, but current-looking output targeting appears to represent rules used in practice reasonably well. We therefore study the rule $\widetilde{R}_t = \phi_\pi \widetilde{\pi}_{t+1} + \phi_y \widetilde{y}_t$. For the simple inflation rule, an active policy results in determinacy unless the degree of distortions in the economy is unreasonably small. Secondly, as we saw before, output targeting supports the implementation of interest rate policy since it reduces the pressure on the monetary authority to react aggressively to inflation deviations. As it turns out, this reasoning applies to the CGG-rule as well.

Theorem 5 *Under a rule with forward-looking inflation and current-looking output targeting equilibrium determinacy obtains if and only if*

$$\max\left\{1-\frac{1-\beta}{\kappa}\phi_y,0\right\}<\phi_\pi<1+2\frac{1+\beta}{\kappa\sigma}+\frac{1+\beta}{\kappa}\phi_y.$$

Proof. See Appendix. ■

The upper and lower bounds on the inflation coefficient reflect the impact of targeting output in addition to inflation. The lower bound coincides with that derived under the current-looking Taylor rule. Again, the feedback effect from output movements along the Phillips-curve allows the monetary authority to be less aggressive than otherwise required. The upper bound is a combination of the one derived under purely expected inflation targeting ($1+2\frac{1+\beta}{\kappa\sigma}$), and the additional leeway generated by output targeting ($\frac{1+\beta}{\kappa}\phi_y$).

We also consider a purely forward-looking rule, where $\widetilde{R}_t=\phi_\pi\widetilde{\pi}_{t+1}+\phi_y\widetilde{y}_{t+1}$. This rule presents more of a theoretical curiosity than one that central banks actually implement.[5] Its determinacy properties are given by

Theorem 6 *Under a purely forward-looking Taylor-rule the equilibrium is unique if and only if*

$$\begin{aligned}0 &\leq \phi_y<\frac{1}{\sigma},\\ \max\left\{1-\frac{1-\beta}{\kappa}\phi_y,0\right\} &< \phi_\pi<1+2\frac{1+\beta}{\kappa\sigma}-\frac{1+\beta}{\kappa}\phi_y.\end{aligned}$$

Proof. See Appendix. ■

Equilibrium determinacy under a purely forward-looking rule requires a new component in the form of an additional restriction on the output coefficient. The central bank's response to output movements cannot be larger than the coefficient of relative risk aversion σ^{-1}. In an economy with relatively risk-averse agents (large σ), the central bank does not enjoy as much leeway as under, say, a CCG-rule due to this tight bound. Since agents are less willing to shift resources intertemporally, and thus the real interest rate is less volatile, the output-inflation channel is less potent in supporting monetary policy. However, there is

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[5] Central banks collect daily data on variables that are fairly reliable form forecasting inflation. This includes, for example, the price of raw materials and producer's goods, often quoted in auction markets on a daily basis. On the other hand, the time series which can be employed to forecast future output are often released with a substantial time lag.

an additional factor at play. The upper bound of the determinacy region for the inflation coefficient ϕ_π depends negatively on the output coefficient. At the maximum value $\phi_y = \sigma^{-1}$ this bound reduces to $1 + \frac{1+\beta}{\kappa\sigma}$ which is less than the corresponding value for pure expected inflation targeting. This upper bound can come arbitrarily close to one if, say, the degree of price stickiness $\varphi \to \infty$. In this case, no unique equilibrium exists, but this is true for all policy rules that include backward- or forward-looking components.

4.3 Backward-Looking Taylor-Rule

Finally, we consider the case when the interest rate is set based on lagged information only, that is, $\widetilde{R}_t = \phi_\pi \widetilde{\pi}_{t-1} + \phi_y \widetilde{y}_{t-1}$.

Theorem 7 *Under a purely backward-looking Taylor-rule, necessary and sufficient conditions for equilibrium determinacy are either*

$$\max\left\{1 - \frac{1-\beta}{\kappa}\phi_y, 0\right\} < \phi_\pi < 1 + 2\frac{1+\beta}{\kappa\sigma} - \frac{1+\beta}{\kappa}\phi_y,$$

for $0 \leq \phi_y < \frac{1+\beta}{\sigma\beta}$*, or*

$$\max\left\{1 + 2\frac{1+\beta}{\kappa\sigma} - \frac{1+\beta}{\kappa}\phi_y, 0\right\} < \phi_\pi < \max\left\{1 - \frac{1-\beta}{\kappa}\phi_y, 0\right\},$$

for $\phi_y > \frac{1+\beta}{\sigma\beta}$.

Proof. See Appendix. ■

As in the previous case, the output coefficient is subject to a parameter restriction. Its only effect, however, is to reverse upper and lower bounds. Note that for sufficiently large ϕ_y, the inflation coefficient ϕ_π can go arbitrarily close to zero. Active output targeting renders inflation targeting moot for the purposes of achieving equilibrium determinacy. To what extent such a policy is optimal or stabilizes business cycles would be worth investigating. Furthermore, a purely backward-looking Taylor rule does not have the same determinacy properties as a purely forward-looking, which was the case for simple inflation targeting. The reason is that responding to output movements introduces an additional channel that imposes different cross-equation restrictions.

5 Interest-Rate Inertia

It has become common wisdom that central bank in many countries try to adjust interest rates only sluggishly. We therefore study another variant of the Taylor rule:

$$\widetilde{R}_t = \phi_R \widetilde{R}_{t-1} + \phi_\pi \widetilde{\pi}_{t+i} + \phi_y \widetilde{y}_t, \tag{26}$$

where $\phi_R \geq 0$ indicates the interest rate smoothing coefficient; $i \in \{0, 1\}$. Empirical estimates suggest a value for ϕ_R around 0.7 which implies a fair degree of persistence (for instance, Clarida et al., 2000, and Rudebusch, 2002). Interest rate inertia can be justified in several ways. The best known argument is probably related to the role of financial markets: interest rate smoothing prevents sudden reactions of financial markets which has been strongly criticized by Svenson (2003) on account of the robustness of the financial sector in advanced economies. Woodford (2003), on the other hand, argues that the presence of a lagged interest rate allows the central bank to infer agents' expectations since it induces history dependence in the monetary policy process. However, Rudebusch (2002) argues that the high coefficient of ϕ_R reported in previous empirical estimates is inconsistent with the historical evidence about the term structure of interest rates (which would include the persistence effect).

In our view, the most coherent rationale behind a rule like (26) is to assign a specific goal to interest rate stabilization. A rule including an interest rate lag can be thought of as an explicit target for monetary authority, like inflation and output, derived from the minimization of a quadratic loss function including interest rate. The interest rate thus becomes an additional target variable, which is separate from the instrument rate, but related to it via the expectation hypothesis of the term structure. Rule (26) and its variants therefore represent a useful starting point.

5.1 Inertia and Current Inflation

We begin by analyzing the determinacy properties of an inertial rule with current inflation and output targeting.

Theorem 8 *Under the inertial Taylor rule (26), necessary and sufficient conditions for a* *unique equilibrium are:*

$$\max\left\{1-\phi_R-\frac{1-\beta}{\kappa}\phi_y,0\right\}<\phi_\pi,$$

$$0\leq\phi_R<\beta.$$

Proof. See Appendix. ■

The determinacy condition reaffirms the Taylor principle: if inflation is above its target level, the interest rate is eventually increased more than proportionally. There is, however, a further qualification: the inertial coefficient ϕ_R has to be less than the discount rate. This stands in contrast to results obtained by Rotemberg and Woodford (1999) who found that it is optimal for the monetary authority to be *super-inertial*, i.e. to set the smoothing coefficient $\phi_R>1$. To conclude, there seems to be a tension between the optimal degree of interest rate inertia and the value of ϕ_R delivering determinacy of the equilibrium: from the optimality point of view, there are no restrictions to the magnitude of ϕ_R.

5.2 Inertia and Expected Inflation

Clarida et al. (2000) suggest the following monetary policy rule as a representation of the Federal Reserve's behavior over the post-war period:

$$\widetilde{R}_t=\phi_R\widetilde{R}_{t-1}+\phi_\pi\widetilde{\pi}_{t+1}+\phi_y\widetilde{y}_t, \tag{27}$$

This equation now includes an expected inflation term and again takes account of the sluggish behavior of interest rates. We establish the determinacy properties of this rule in the following

Theorem 9 *Under the forward looking rule with interest rate smoothing (27) necessary and sufficient conditions for a unique equilibrium are given by:*

$$\max\left\{1-\phi_R-\frac{1-\beta}{\kappa}\phi_y,0\right\}<\phi_\pi, \tag{28}$$

$$\phi_\pi<1+2\frac{1+\beta}{\kappa\sigma}+\frac{1+\beta}{\kappa}\phi_y+\left(1+2\frac{1+\beta}{\kappa\sigma}\right)\phi_R \tag{29}$$

and

$$0\leq\phi_R<\beta.$$

Proof. See Appendix. ■

The lower bound is identical to the one derived above, as is the restriction on ϕ_R. The two inertial rules differ, however, in their upper bounds for the inflation coefficient. These results echo a theme from above in that adding target variables expands the range of determinate inflation coefficients, although the expected inflation component imposes an upper bound.[6] Secondly, even a moderately large smoothing coefficient implies that ϕ_π can be well below unity. Interest rate inertia perpetuates even small interest rate movements, which forward-looking agents take into account. The logic behind this result lies with the long run impact coefficient of the inflation rate: with $\phi_R > 1$ the long run impact coefficients on the inflation rate is smaller than for $\phi_R < 1$. To get the same control on the inflation rate that can be achieved with a small value, ϕ_R would have to be set equal to a very large number.

6 Summary and Conclusion

A summary of this paper is simple: Inflation targeting works! A monetary authority that is concerned about introducing instability in an economy can safely adhere to a policy rule that raises the real interest rate in response to inflationary pressures. In the simple inflation-targeting framework, this implies setting the nominal rate such that it moves more than proportionally than the inflation target. If the central bank's objectives include concern for output the range of stabilizing policy expands up to the point where not even an aggressive anti-inflationary stance is required. A similar conclusion also carries over to a policy of interest-rate smoothing. While authors such as Carlstrom and Fuerst (2000) express a certain degree of determinacy pessimism, we do not find this warranted. Irrespective of the central bank's targets there is an interest rate rule that guarantees equilibrium determinacy if it is sufficiently responsive to economic conditions.

This does point, however, to a potential shortcoming of this, and other, research on the determinacy properties of policy rules: recommendations are conditional on the modeling framework - and often strikingly so. For instance, Carlstrom and Fuerst (2001) show that a subtly different timing convention for money holdings from the one used in this paper,

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[6] It can be quickly verified that for parameter values commonly encountered in the literature the upper bound is far above the range of reasonable inflation coefficients.

and most monetary models, implies different determinacy properties. Along the same line, introducing investment also affects determinacy. In such a framework, the real rate of return on capital provides an additional transmission and feedback mechanism for monetary policy. Carlstrom and Fuerst (2000) and Lubik (2003) show how the standard intuition of stabilizing anti-inflationary monetary policy no longer applies. Moreover, in a seminal paper Leeper (1991) has demonstrated that analysis of monetary policy cannot be separated from fiscal policy. The discussion in this paper implicitly assumes that fiscal policy is passive and accommodates intertemporal budget balance. A more independent fiscal policy therefore changes the determinacy properties of monetary rules.

Where do we go from here? It seems obvious and redundant to suggest that economists need to develop a better understanding of the macroeconomy. However, at the very least they should be more careful and aware in using models for policy analysis that can deliver markedly different recommendations at the change of seemingly small details. Ultimately however, this is an empirical question. Our work may prove useful for guiding researchers in the empirical analysis of monetary policy models that allow for multiple equilibria. Since aggregate dynamics under determinacy and indeterminacy are potentially different, empirical analysis needs to take this into account such as in Lubik and Schorfheide (2003).

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Appendix

A Schur Cohn Criterion

The characteristic equation of a 2×2 matrix A is given by $x^2 - tr(A)x + \det(A) = 0$. It is well known that the condition for two roots of the characteristic equation to lie outside the unit circle is (see LaSalle, 1986):

$$|\det(A)| > 1, \tag{30}$$

$$|tr(A)| < 1 + \det(A). \tag{31}$$

In the models under consideration we also encounter 3×3 matrices with the associated characteristic, third-order polynomial equation:

$$\lambda_3 + A_2\lambda^2 + A_1\lambda + A_0 = 0.$$

Woodford (2003) derives necessary and sufficient conditions for two roots to lie outside and one root within the unit circle. To wit, the following restrictions have to hold: Either:

- Case I:

$$\begin{aligned} 1 + A_2 + A_1 + A_0 &< 0, \quad (A.1) \\ -1 + A_2 - A_1 + A_0 &> 0, \quad (A.2) \end{aligned}$$

or

- Case II:

$$\begin{aligned} 1 + A_2 + A_1 + A_0 &> 0, \quad (A.3) \\ -1 + A_2 - A_1 + A_0 &< 0, \quad (A.4) \\ A_0^2 - A_0 A_2 + A_1 - 1 &> 0, \quad (A.5) \end{aligned}$$

or

- Case III:

$$\begin{aligned} 1 + A_2 + A_1 + A_0 &> 0, \quad (A.3) \\ -1 + A_2 - A_1 + A_0 &< 0, \quad (A.4) \\ A_0^2 - A_0 A_2 + A_1 - 1 &< 0, \quad (A.6) \\ |A_2| &> 3. \quad (A.7) \end{aligned}$$

B Proofs

B.1 Proof of Theorem 4

Determinacy requires two explosive roots, the conditions of which are given in the Appendix. The system matrix is:

$$\begin{bmatrix} 1/\beta & -\frac{\kappa}{\beta} \\ \sigma\left(\phi_\pi - \frac{1}{\beta}\right) & 1 + \frac{\kappa\sigma}{\beta} + \sigma\phi_y \end{bmatrix}.$$

Its trace and determinant are given by, respectively: $tr = 1 + \frac{1}{\beta} + \frac{\kappa\sigma}{\beta} + \sigma\phi_y$ and $\det = \frac{1}{\beta} + \frac{\sigma}{\beta}\left(\kappa\phi_\pi + \phi_y\right)$. Over the admissible parameter range, the determinant is always strictly above one, so that Condition 1 holds. It can be quickly verified that the right-hand-side of Condition 2 implies $\phi_\pi > 1 - \frac{1-\beta}{\kappa}\phi_y$, while the left-hand-side leads to $\phi_\pi > 0 > -1 - 2\frac{1+\beta}{\kappa\sigma} - \frac{1-\beta}{\kappa}\phi_y$. The Theorem follows immediately.

B.2 Proof of Theorem 5

The system matrix is:

$$\left[\begin{array}{cc} 1/\beta & -\frac{\kappa}{\beta} \\ \frac{\sigma}{\beta}\left(\phi_\pi - 1\right) & 1 + \sigma\phi_y - \frac{\kappa\sigma}{\beta}\left(\phi_\pi - 1\right) \end{array}\right].$$

Its trace and determinant are given by, respectively: $tr = 1 + \frac{1}{\beta} + \sigma\phi_y - \frac{\kappa\sigma}{\beta}\left(\phi_\pi - 1\right)$ and $\det = \frac{1}{\beta} + \frac{\sigma}{\beta}\phi_y$. Over the admissible parameter range, the determinant is always strictly above one, so that Condition 1 holds. The right-hand side inequality of Condition 2 implies that $\phi_\pi > 1 - \frac{1-\beta}{\kappa}\phi_y$, while the left-hand-side leads to $\phi_\pi < 1 + 2\frac{1+\beta}{\kappa\sigma} + \frac{1+\beta}{\kappa}\phi_y$. The Theorem then follows immediately.

B.3 Proof of Theorem 6

The coefficient matrix is:

$$\left[\begin{array}{cc} \frac{1}{\beta} & -\frac{\kappa}{\beta} \\ -\frac{\sigma}{\beta}\frac{1-\phi_\pi}{1-\sigma\phi_y} & \frac{1+\frac{\kappa\sigma}{\beta}(1-\phi_\pi)}{1-\sigma\phi_y} \end{array}\right].$$

Its trace and determinant are given by, respectively: $tr = \frac{1}{\beta} + \frac{1+\frac{\kappa\sigma}{\beta}(1-\phi_\pi)}{1-\sigma\phi_y}$ and $\det = \frac{1}{\beta\left(1-\sigma\phi_y\right)}$. We prove the Theorem by first deriving policy parameter restrictions from the condition $|\det| > 1$. We start checking for parameter combinations such that $\det > 1$. We have to distinguish two cases. For $\phi_y < \sigma^{-1}$ we have $\phi_y > 0 > -\frac{1}{\sigma}\frac{1-\beta}{\beta}$, whereas for $\phi_y > \sigma^{-1}$, $\phi_y < -\frac{1}{\sigma}\frac{1-\beta}{\beta} < 0$, which is ruled out by assumption. Consequently, a first restriction is given by $0 \leq \phi_y < \sigma^{-1}$. Next, we consider $\det < -1$. The two cases lead to the conditions, respectively, $\phi_y > \frac{1}{\sigma}\frac{1-\beta}{\beta} > \sigma^{-1}$ and $\phi_y < \frac{1}{\sigma}\frac{1-\beta}{\beta} < \sigma^{-1}$. The first condition is inconsistent, while the second condition provides an additional range for which $|\det| > 1$ is true. The parameter restriction derived from this is therefore $0 \leq \phi_y < \frac{1}{\sigma}\frac{1-\beta}{\beta}$.

Next, we derive restrictions from the condition $tr < 1 + \det$. For $\phi_y < \sigma^{-1}$ this holds if $\phi_\pi > 1 - \frac{1-\beta}{\kappa}\phi_y$, whereas $\phi_y > \sigma^{-1}$ implies $\phi_\pi < 1 - \frac{1-\beta}{\kappa}\phi_y$. Similarly, the condition $-\left(1 + \det\right) < tr$ implies $\phi_\pi < (>) 1 + 2\frac{1+\beta}{\kappa\sigma} - \frac{1-\beta}{\kappa}\phi_y$ if $\phi_y < (>) \sigma^{-1}$. We now combine the restrictions derived from the two conditions. If $0 \leq \phi_y < \sigma^{-1}$, we have $1 - \frac{1-\beta}{\kappa}\phi_y < \phi_\pi < 1 + 2\frac{1+\beta}{\kappa\sigma} - \frac{1-\beta}{\kappa}\phi_y$. This range is non-empty if $\phi_y < \frac{1}{\sigma}\frac{1+\beta}{\beta}$, which is always true under the initial assumption $\phi_y < \sigma^{-1}$. If $\sigma^{-1} < \phi_y < \frac{1}{\sigma}\frac{1-\beta}{\beta}$ we find $1 + 2\frac{1+\beta}{\kappa\sigma} - \frac{1-\beta}{\kappa}\phi_y < \phi_\pi < 1 - \frac{1-\beta}{\kappa}\phi_y$, which is non-empty if $\phi_y > \frac{1}{\sigma}\frac{1+\beta}{\beta}$. This condition, however, violates the initial assumption. The Theorem then follows immediately.

B.4 Proof of Theorem 7

The system matrix is given by:

$$\left[\begin{array}{ccc} 1/\beta & -\frac{\kappa}{\beta} & 0 \\ -\frac{\sigma}{\beta} & 1+\frac{\kappa\sigma}{\beta} & \sigma \\ \phi_\pi & \phi_y & 0 \end{array}\right],$$

with the associated characteristic equation: $\lambda_3 - \left(1+\frac{1}{\beta}+\frac{\kappa\sigma}{\beta}\right)\lambda^2 + \left(\frac{1}{\beta}-\sigma\phi_y\right)\lambda + \frac{\sigma}{\beta}\left(\kappa\phi_\pi+\phi_y\right) = 0$. The system contains two unstable and one stable root. We therefore apply the conditions listed in Appendix A. Condition (A.1) of Case I leads to $\phi_\pi < 1-\frac{1-\beta}{\kappa}\phi_y = \phi_\pi^U$, while (A.2) implies $\phi_\pi > 1+2\frac{1+\beta}{\kappa\sigma}-\frac{1+\beta}{\kappa}\phi_y = \phi_\pi^L$. We now check if the two restrictions are mutually exclusive. Simple algebra shows that $\phi_\pi^L < \phi_\pi^U$ iff $\phi_y > \frac{1+\beta}{\sigma}$, which proves the second part of the Theorem.

Conditions (A.3) and (A.4) of Case II lead to $\phi_\pi > 1-\frac{1-\beta}{\kappa}\phi_y$ and $\phi_\pi < 1+2\frac{1+\beta}{\kappa\sigma}-\frac{1+\beta}{\kappa}\phi_y$, respectively. Condition (A.5) requires

$$\left(1+\beta^{-1}+\frac{\kappa\sigma}{\beta}\right)^2 + \frac{\sigma}{\beta}\left(1+\beta^{-1}+\frac{\kappa\sigma}{\beta}\right)\left(\kappa\phi_\pi+\phi_y\right) + \beta^{-1} - \sigma\phi_y - 1 > 0,$$

which can be rewritten as

$$\left(1+\beta^{-1}+\frac{\kappa\sigma}{\beta}\right)^2 + \frac{1-\beta}{\beta} + \frac{\kappa\sigma}{\beta}\left(1+\beta^{-1}+\frac{\kappa\sigma}{\beta}\right)\phi_\pi > -\frac{\sigma}{\beta}\left(1+\beta^{-1}+\frac{\kappa\sigma}{\beta}-\beta\right)\phi_y.$$

Since the coefficient term on the right-hand side is non-negative, this restriction always holds. Case III is therefore redundant. It remains to show that the parameter range derived under Case II is non-empty. This is the case iff $\phi_y < \frac{1+\beta}{\sigma}$. The first part of the Theorem follows immediately.

B.5 Proof of Theorem 8

After substituting out rule (26) into the system formed by (19)-(20) and rearranging, we can define $\widetilde{x}_t = \left[\widetilde{\pi}_t, \widetilde{y}_t, \widetilde{R}_{t-1}\right]'$ so that matrix A in (**??**) is defined as:

$$A = \left[\begin{array}{ccc} \beta^{-1} & -\kappa\beta^{-1} & 0 \\ \sigma\phi_\pi - \sigma\beta^{-1} & \sigma\kappa\beta^{-1}+1+\sigma\phi_y & \sigma\phi_R \\ \phi_\pi & \phi_y & \phi_R \end{array}\right]$$

The characteristic polynomial of matrix A is given by $p(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$, where:

$$\begin{aligned} a_2 &= -\left[1 + \frac{1}{\beta} + \frac{\sigma\kappa}{\beta} + \phi_R + \sigma\phi_y\right] \\ a_1 &= \frac{\sigma\kappa}{\beta}(\phi_\pi + \phi_R) + \phi_R + \frac{\phi_R}{\beta} + \frac{\sigma\phi_y}{\beta} + \frac{1}{\beta} \\ a_0 &= -\frac{\phi_R}{\beta} \end{aligned}$$

to get determinacy, we need two roots outside the unit circle and one inside, since two variables are jump variables, and one is predetermined. Let us start by examining the Schur-Cohn conditions. Starting with Case I, from (A.1) we find $\kappa\left(\phi_\pi + \phi_y - 1\right) + \phi_y(1-\beta) < 0$, while condition (A.2) is never verified, given the assumptions made on the coefficients a_2, a_1, a_0, and on ϕ_π, ϕ_y, ϕ_R being all positive. This is enough to exclude Case I.

According to Case II, we have that condition (A.3) immediately leads to the second condition, and condition (A.4) is always verified. Finally, from condition (A.5) we find:

$$\phi_R^2\left(\frac{1-\beta}{\beta}\right) + \sigma\phi_y(1-\phi_R) - \sigma\kappa\phi_R\frac{(1-\beta)}{\beta} + \sigma\kappa\phi_\pi + \beta\phi_R - \frac{\phi_R}{\beta} + 1 - \beta > 0 \tag{32}$$

Adding and subtracting from the above expression ϕ_R and $\sigma\kappa$ we get:

$$\sigma\kappa(\phi_\pi + \phi_R - 1) + \sigma\phi_y(1-\phi_R) + \frac{(1-\beta)(1-\phi_R)(\beta-\phi_R)}{\beta\sigma\kappa} + \frac{\sigma\kappa(\beta-\phi_R)}{\beta} > 0 \tag{33}$$

which is immediately satisfied if $\phi_\pi + \phi_R > 1$ and if $\phi_R < \beta$. In particular, from (33) we can derive another bound for ϕ_π:

$$\phi_\pi > (1-\phi_R)\left(1 - \frac{\sigma\phi_y}{\sigma\kappa}\right) - \frac{(\beta-\phi_R)}{\beta}\left[1 + \frac{(1-\beta)(1-\phi_R)}{\sigma\kappa}\right] \tag{34}$$

It is immediate to check that if $\phi_\pi + \phi_R > 1$ and $\phi_R < \beta$, the bound implied by (34) bigger than that given by the second condition. Therefore, when this is true and $\phi_R < \beta$, then (34) is automatically satisfied.

To conclude the proof we need to show that condition under Case III do not apply in this context. First of all, under Case III, the second condition and (A.4) have to hold, exactly as for Case II. Moreover condition (A.6) implies equation (33) with the inequality sign reverted, together with condition (A.7) which delivers the following restriction:

$$\sigma\phi_y + \phi_R + \frac{1+\sigma\kappa}{\beta} > 2$$

It is immediate to check that in order to satisfy (A.6) we need to set at least $\phi_R > 1$ together with $\phi_\pi + \phi_R < 1$: this last restriction violatesthe second condition. This is enough to exclude Case III.

B.6 Proof of Theorem 9

After substituting out the rule in the system formed by (19)-(20) and rearranging, we can define $\widetilde{x}_t = \left[\widetilde{\pi}_t, \widetilde{y}_t, \widetilde{R}_{t-1}\right]'$ so that matrix A is given by:

$$A = \begin{bmatrix} 1/\beta & -\kappa/\beta & 0 \\ \beta^{-1}\sigma\left(\phi_\pi - 1\right) & -\beta^{-1}\sigma\kappa\left(\phi_\pi - 1\right) + 1 + \sigma\phi_y & \sigma\phi_R \\ \beta^{-1}\phi_\pi & -\beta^{-1}\kappa\phi_\pi + \phi_y & \phi_R \end{bmatrix}$$

The characteristic polynomial of matrix A is given by: $p(\lambda) = \lambda^3 + a_2\lambda^2 + a_1\lambda + a_0$, where:

$$\begin{aligned} a_2 &= -\left[1 + \frac{1}{\beta} - \frac{\sigma\kappa}{\beta}\left(\phi_\pi - 1\right) + \phi_R + \sigma\phi_y\right] \\ a_1 &= \frac{\sigma}{\beta}\left(\kappa\phi_\pi + \phi_y\right) + \phi_R + \frac{\phi_R}{\beta} + \frac{1}{\beta} \\ a_0 &= -\frac{\phi_R}{\beta} \end{aligned}$$

To get determinacy we need one root inside the unit circle and two roots outside, because R_t is the only predetermined variable, while (π_t, Y_t) are jump variables. In order to prove the result, let us start by considering Case I in Appendix A. From condition (A.1) we get: $k\left(\phi_\pi + \phi_y - 1\right) + \phi_y\left(1 - \beta\right) < 0$. This implies that the upper bound for ϕ_π is given by:

$$\phi_\pi < 1 - \phi_R - \frac{\phi_y\left(1 - \beta\right)}{\kappa} \tag{35}$$

However, condition (A.2) implies a lower bound for ϕ_π, given by:

$$\phi_\pi > 1 + \frac{(1 + \beta)}{\sigma\kappa}\left(2 + \sigma\phi_y\right) + \frac{\phi_R}{\sigma\kappa}\left[2\left(1 + \beta\right) + \sigma\kappa\right]$$

a contradiction with (35): this is enough to exclude Case I.

Under Case II, condition (A.3) directly implies the lower bound, while (A.4) leads immediately to the upper bound. From (A.5), after simplification, we get:

$$\frac{\phi_R^2}{\beta^2} - \frac{\phi_R}{\beta^2} - \frac{\sigma\phi_y\phi_R}{\beta} - \frac{\phi_R^2}{\beta} + \frac{\sigma\kappa\phi_R\left(\phi_\pi - 1\right)}{\beta^2} + \frac{1}{\beta} + \frac{\sigma\phi_y}{\beta} + \phi_R + \frac{\sigma\kappa\phi_R}{\beta} - 1 > 0 \tag{36}$$

By adding and subtracting ϕ_R and $\sigma\kappa\phi_R\beta^{-1}$ from (36) we get:

$$\frac{\sigma\kappa\phi_R}{\beta}\left(\phi_\pi + \phi_R - 1\right) + \sigma\phi_y\left(1 - \phi_R\right) + \frac{(1 - \beta)\left(1 - \phi_R\right)\left(\beta - \phi_R\right)}{\beta} + \frac{\sigma\kappa\phi_R\left(\beta - \phi_R\right)}{\beta} > 0 \tag{37}$$

which is satisfied if $\phi_\pi + \phi_R > 1$ (this is directly implied by the lower bound) and $0 \leq \phi_R < \beta$. Note that condition (37) implies the following lower bound for ϕ_π :

$$\phi_\pi > (1 - \phi_R)\left(1 - \frac{\phi_y \beta}{\kappa \phi_R}\right) - (\beta - \phi_R)\left(1 + \frac{(1-\beta)(1-\phi_R)}{\sigma \kappa \phi_R}\right)$$

which is automatically respected given the lower bound and $0 \leq \phi_R < \beta$.

To complete the proof we are going to show that the conditions under Case III do not apply in this context. Conditions (A.3) and (A.4) of Case III imply the upper and lower bounds, respectively. At the same time, condition (A.6) implies (37) with the inequality sign reverted: the resulting inequality is satisfied if $\phi_\pi + \phi_R < 1$ and $\phi_R > 1$. Evidently, the requirement $\phi_\pi + \phi_R < 1$ violates (A.3), and this allows us to exclude Case III.