Durable-Goods Oligopoly with Secondary Markets: Theory and an Empirical Application to the Automobile Market*

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Abstract

We examine the effects of durability on equilibrium producer behavior in the car market. In this setting, forward-looking producers take into account the effect that their current production decisions have on their current and future profits, due to the existence of a secondary market. First, we construct a dynamic oligopoly model of a vertically-differentiated product market to understand the equilibrium production dynamics which arise from the durability of the goods and their active trade in secondary markets. Second, we use data from the automobile industry to estimate a tractable linear-quadratic version of this model. One result suggests that durability may be a particularly desirable car feature for high-quality car producers since, by overproducing today, they can exploit durability and the existence of a secondary market to potentially reduce their lower-quality competitors' future production: planned obsolescence appears to be a more profitable strategy for lower-end than higher-end producers.

1 Introduction

In many durable goods industries, used products are traded in decentralized secondary markets which are not directly controlled by the producers of new goods: the automobile industry is perhaps the most prominent example. In this paper we seek to understand the effects of durability and secondary markets on equilibrium behavior in this industry. In the context of a dynamic equilibrium model, we quantify explicitly how product durability and trade in secondary markets affects equilibrium producer behavior in the automobile market.

The durability of cars and the existence of a secondary market have important competitive implica-

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tions for new car producers.[1] Obviously, the secondary market introduces, in the form of used cars, a large number of (imperfect) substitutes to the new cars produced each period, limiting the market power of each producer. However, this detrimental effect on firms' market power is mitigated by the ability of consumers to trade cars in a secondary market, which introduces an additional component — the resale value — to consumers' valuations of new cars. New cars become (in part) investment assets, and this investment motive may raise consumers' willingness-to-pay for new cars.

Furthermore, rational firms recognize that current production will reach the secondary market in the future and, by lowering prices in those markets, erodes both present as well as future profits. A durable goods monopolist fully internalizes this effect by curtailing current production. In a durable goods oligopoly, however, each producer internalizes only the effect this has on his own future profits, but not the detrimental effect it has on its rival's future profits. Indeed, each oligopolistic producer derives an indirect benefit from increases in current production if this causes its rivals to lower their future production levels; in equilibrium, therefore, a firm may choose to overproduce today if these indirect benefits outweigh the costs of more vigorous competition from the secondary market tomorrow.

We make two contributions in this paper. First, we construct a dynamic oligopoly model of a vertically-differentiated product market to understand the equilibrium production dynamics which arise from the durability of the goods and their active trade in secondary markets. Second, we use data from the automobile market to estimate a tractable linear-quadratic version of the model. To our knowledge, this paper is the first empirical study of the car market within the framework of an equilibrium dynamic oligopoly model which recognizes the intertemporal linkages resulting from durability and trade in secondary markets. Our results shed light on the competitive effects of durability in this market, and we are able to quantify directly the influence of secondary markets and durability on equilibrium production.

One important insight from our results is that producers of high quality goods may benefit more from durability than lower-quality producers, implying that planned obsolescence may be more profitable for lower-end than higher-end producers. A high-end producer, by producing more today, intensifies the future competition at the lower end of the market as these cars age, which can reduce the future production of the low-end producers. However, this effect is asymmetric: since low-end producers cannot affect their high-end rivals in the same way, they may prefer to make their cars less durable in order to reduce competition with their past production in the secondary market. Perhaps this explains the absence of durable cars at the lower-end, and nondurable cars at the higher-end, of the car market.

[1] The Alcoa antitrust case inspired much academic interest in a similar question: the competitive implications of a primary market and a recycling industry where used aluminum can be reprocessed; see, for example, Gaskins (1974).

1.1 Background and existing literature

Since the seminal work of Coase (1972), there has been a large theoretical literature analyzing how durability erodes market power for a monopoly producer. Coase conjectured that a monopoly producer of an infinitely-durable good may lose all of his market power due to his inability to commit to low levels of future production.[2] Subsequent work (eg. Stokey (1981), Gul, Sonnenschein, and Wilson (1986), and Ausubel and Deneckere (1989)) confirmed Coase's conjecture as an equilibrium limiting result in models where the time lag between the monopolist's price offers shrinks to zero.[3]

We use our estimates to simulate counterfactual experiments which illustrate how firms' production behavior would differ were they able to commit to future production paths. These simulations suggest that the ability to commit would raise firms' profits, thus supporting Coase's conjecture. In the monopoly setting which has been the focus of most of the Coase Conjecture literature, these higher profits can only be achieved by decreases in production. In a durable goods oligopoly, however, this is not necessarily true: our results indicate that some firms would obtain higher profit levels by *increasing* their equilibrium production levels.

In the presence of commitment problems, a secondary market may sharpen firms' incentives to hold back current production (thus counteracting the Coasian tendencies) because, if used cars are substitutes for new cars, any overproduction today will lead to lower used car prices (and therefore lower new car sales) in the future.[4] However, even with secondary markets, a fundamental commitment problem still exists, since producers would like to commit to low future production levels in order to increase future resale values and, thereby, raise consumers' willingness-to-pay for new cars today. Bulow (1986) shows that, generally, a monopolist would try to reduce this commitment problem by planned obsolescence; that is, by reducing the durability of its product.[5] However, in a quantity-setting oligopoly, each producer has a countervailing incentive to increase durability, in order to reduce its rivals' future production.

While we do not endogenize durability choice in this paper, we use our estimates to simulate counterfactuals to gauge how the profitability of obsolescence varies across firms. We find that, with vertically-differentiated durable goods, this countervailing incentive is stronger for firms producing higher-quality goods, since used variants of these goods substitute readily with the new goods produced by lower-quality firms. One striking implication of this finding is that, in vertically-differentiated durable-goods oligopolies, the profitability of planned obsolescence varies across firms

[2]See Bulow (1982) for a treatment of the durable-goods monopolist problem within a two-period model.

[3]However, Ausubel and Deneckere (1989) also prove a Folk Theorem for the limiting durable-goods monopoly game, and show that the Coase outcome is but one of a continuum of subgame-perfect equilibria for this game, which also includes an equilibrium in which the producer obtains profits arbitrarily close to the full-commitment monopoly profits.

[4] See Liang (1999) for an articulation of this idea for a monopolist. Furthermore, results for a homogeneous product durable goods oligopoly from Ausubel and Deneckere (1987) and Gul (1987) show that, somewhat paradoxically, oligopolistic competition also acts as a commitment device which allows firms to avoid the Coase outcome by strengthening incentives to collude. The strategies which sustain collusive equilibria fall outside the linear-quadratic framework which we focus on in this paper.

[5]The issue of planned obsolescence has sparked a large theoretical literature; see, for example, Swan (1985) and Rust (1985b).

at different ends of the quality spectrum.

The implications of durability and secondary markets on the dynamics of car demand have not been ignored in the literature. Berkovec (1985), Rust (1985a), and Stolyarov (2000) focus on dynamic consumer demand in a durable goods market with primary, secondary and scrappage market segments, but assume a simplified supply side in which primary market prices evolve exogenously. Adda and Cooper (2000) employ the optimal decision rules from a dynamic discrete-choice model to explore the effects of scrappage subsidies on car demand, where cars are held until scrapped and, hence, are not actively traded in the secondary market.[6] The two closest antecedents to our paper are Porter and Sattler (1999), who test empirical predictions on the volume of trade in secondary car markets using a durable-goods monopoly model with transactions costs and full commitment, and Esteban (2001), who introduces a dynamic model of semi-durability and imperfect competition to characterize the equilibrium dynamics in prices and production. These papers do not consider differentiation (in both quality as well as durability) among the car models produced by oligopolistic rivals, which we focus on in this paper.

To our knowledge, we are among the first to undertake an empirical analysis of the car market which accommodates the equilibrium production dynamics arising from car durability and active secondary markets. Empirically addressing the equilibrium effects of durability in a durable goods oligopoly is a challenging task because it requires that we derive suppliers' equilibrium production rules. Since durable goods markets are characterized by both forward-looking consumers as well as producers, it is generally difficult to describe a dynamic equilibrium in such a market. For example, forward-looking consumers base their current purchase decisions on their beliefs regarding future prices and/or quantities; however, a consistency requirement (such as rational expectations) implies that these beliefs must indeed be consistent with the firms' actual pricing and/or production decisions in equilibrium. Symmetrically, firms' pricing and production decisions must be profit-maximizing best-responses, given consumers' beliefs.

In this paper, we overcome these challenges by constructing a dynamic equilibrium model of the car market in which tractability and flexibility is provided by its linear-quadratic structure.[7] This model captures four key characteristics of the car industry: oligopolistic automobile producers, an active, decentralized secondary market,[8] quality differentials between the car models, and differing quality depreciation schedules.

[6]Attanasio (2000) considers an (S, s) model in which idiosyncratic shocks lead consumers to not only upgrade, but also downgrade their car stock. See also Melnikov (2000) for an empirical analysis of demand dynamics in the computer printer market, where durability and quality improvements create an option value to delaying purchases.

[7]See Kydland (1975) for a description of discrete-time linear-quadratic dynamic games, and Judd (1996) for an application of linear-quadratic models to a dynamic oligopoly setting where firms set both prices and quantities. Kahn (1986) analyzes the effects of increasing costs in an infinitely-durable goods monopoly framework and applies a linear-quadratic structure.

[8]Beginning with Akerlof (1970), one strand of the literature on secondary markets has focused at informational issues; recent work by Hendel and Lizzeri (1999a) consider a two-period durable goods market, and House and Leahy (2000) extend this framework to a longer time horizon. In this article, we assume a world of perfect information, and so abstract from adverse selection issues.

Our emphasis on the dynamics due to durability and secondary markets distinguishes our work from existing empirical studies of the automobile industry. Bresnahan (1981), Berry, Levinsohn, and Pakes (1995), Goldberg (1995), and Petrin (1999) have employed static models of demand and supply in order to estimate the degree of market power and the welfare effects of new product introduction in the car industry.

We structure the article as follows. In Section 2 we introduce the model, and derive the Markov Perfect Equilibrium of the dynamic game. Subsequently, we derive a linear-quadratic specification of this model which is convenient for the empirical work. In Section 3 we describe the data and discuss the empirical implementation of the model. In Section 4 we describe our estimation results and conduct different simulations. We conclude in Section 5.

2 A Model of Durable Goods Oligopoly with Secondary Markets

We consider a dynamic quantity-setting game among oligopolistic producers of differentiated durable goods (in this case automobiles). On the demand side, we assume that consumers are forward looking, so that durability and secondary markets introduce dynamic investment considerations into their car consumption decisions. On the supply side, we assume that new car producers are quantity-setting oligopolists who recognize both the intertemporal effect of current production on future profits due to the secondary market as well as the dependence of current profits on past, present and expected future production.[9]

Following Esteban (1999), we assume that the available cars are vertically differentiated. We label cars in their first period of life as *new* cars and, thereafter, as *used* cars. Throughout, we assume that used cars are transacted in competitive and decentralized secondary markets, so that new car producers can manipulate market outcomes in the secondary market only indirectly, through their production of new cars.

Each period, N firms produce new cars. We let $\mathcal{N}$ denote the set of firms, where $N \equiv |\mathcal{N}|$. Each firm $j \in \mathcal{N}$ produces L_j distinct models, where $L_j \equiv |\mathcal{L}_j|$ and $\mathcal{L}_j$ is the set of all models produced by this firm. Then, $\mathcal{L} \equiv \cup_j \mathcal{L}_j$ is the set of all models produced by all firms and $L \equiv |\mathcal{L}|$ is their total number. We index models by $i = 1, \ldots, L$.

New cars differ in quality and durability. For each model $i \in \mathcal{L}$, let $q_{i,h}$ denote its quality at age h, where $h = 1, \ldots, T_i$ indexes its age and $T_i < \infty$ denotes the number of periods it lasts.[10] Then,

[9] We differ from much of the preceding empirical literature on the car industry by assuming that car producers play a quantity-setting rather than price-setting game. Several institutional features help justify this assumption. First, an implicit assumption of the Bertrand price-setting model is flexible capacity, and capacity does not generally appear easily adjustable in car production. Second, in the car market prices seem to adjust to clear the market at given quantity levels, as in the quantity-setting case. For example, rebates are a common way of adjusting new car prices to clear the inventories at the end of the model year. Furthermore, dealer behavior limits the manufacturers' ability to control prices. Finally, the detailed production figures reported in industry trade publications (such as *Ward's*) suggest that, from the producers' perspectives, quantities are the relevant strategic variables (and also the basis of collusion? See Snyder and Doyle (1999)).

[10] Hence, the durability is deterministic.

each car (new or used) is completely described by the tuple (i, h), and the set of all distinct cars transacted is given by $\mathcal{K} \equiv \{(i, h) | i \in \mathcal{L}, h = 1, \ldots, T_i\}$ and $K \equiv |\mathcal{K}|$ is their total number.

Next, we define a mapping $\omega : \mathcal{K} \mapsto \{1, \ldots, K\}$ which ranks cars from highest to lowest quality as follows

$$\forall (i, h) \neq (i', h') \in \mathcal{K}, \; q_{i,h} > q_{i',h'} \Rightarrow \omega(i, h) < \omega(i', h').$$

Hence, a ranking of 1 denotes the highest-quality car, and a ranking of K the lowest quality. Given this ranking, we define a quality ladder as follows.

Definition 1 *A vector $\boldsymbol{\alpha} = [\alpha_1, \; \alpha_2, \; \ldots, \; \alpha_K, 0]'$ is a **quality ladder** representing the quality structure of this problem if $\alpha_k \equiv \{q_{i,h} | k = \omega(i, h)\}$.*

Given a quality ladder $\boldsymbol{\alpha}$, we define a second mapping $\upsilon : \{1, \ldots, K\} \mapsto \{1, \ldots, K\}$ which tracks the position that a car currently in position k occupies after one period of depreciation. Then, $\upsilon(\omega(i, 1)) = \omega(i, 2)$ and, more generally,

$$\upsilon^{h-1}(\omega(i, 1)) \equiv \underbrace{\upsilon(\upsilon(\ldots \upsilon(}_{h-1 \text{ times}}\omega(i, 1)))) = \omega(i, h), \quad \text{for } h = 2, \ldots, T_i - 1.$$

Cars which die (i.e., all cars (i, h) where $h > T_i$) are given a ranking of $K + 1$ (since $\alpha_{K+1} = 0$), so that $\upsilon^{T_i}(\omega(i, 1)) = K + 1$, for all $i \in \mathcal{L}$, and $\upsilon(K + 1) = K + 1$.

Finally, we define $\eta(i) \equiv \omega(i, 1)$ for all $i \in \mathcal{L}$, to simplify notation. Then, we write succinctly the positions in the ladder occupied successively by a model i car as

$$\eta(i), \upsilon(\eta(i)), \upsilon^2(\eta(i)), \ldots \upsilon^{T_i}(\eta(i)).$$

We now turn to the demand side of our model.

2.1 Demand in a vertically differentiated market

2.1.1 The consumer's problem

Our consumer population is a continuum of infinitely-lived agents in which population heterogeneity in consumers' taste for quality generates demand for each type of car. In each period t, each consumer determines her optimal consumption choice among the K available cars and the option of not consuming a car at all (which we index $K + 1$), to maximize her discounted utility function.[11]

For tractability, we assume that consumers face no transactions costs in any primary or secondary market. In deriving consumers' optimal car choice rules, we also assume that they possess perfect foresight about the sequences of future prices across all car markets $p^k_{t+\tau}$ for $k = 1, \ldots, K$, and

[11] In the empirical implementation section, we will interpret the nonpurchase choice as the consumption of an outside good.

$\tau = 1, \ldots, \infty$.[12] We assume the heterogeneity among consumers is parameterized by a scalar type $\theta \in [0, \bar{\theta}]$ (where $\bar{\theta} < \infty$), and is distributed in the population according to the cumulative distribution $F(\cdot)$. The population has size M.

Then, a consumer of type θ chooses a sequence of car choices to maximize her discounted lifetime utility[13]

$$U^{\theta} \equiv \sum_{t=1}^{\infty} \delta^{t-1} U_t^{\theta}, \tag{1}$$

where her period t utility flow is assumed to be quasilinear in income and given by

$$U_t^{\theta} \equiv \alpha_k \theta + m_t^{\theta} - p_t^k, \tag{2}$$

where δ denotes the discount factor common across all consumers and firms, θ measures this consumer's preference for quality, and m_t^{θ} denotes consumer θ's income at the beginning of period t.[14]

These preferences imply that, if all cars were priced identically, all consumers would choose the highest quality car; hence, cars are *vertically differentiated.*[15] As in all vertically-differentiated markets, both heterogeneity in quality across cars as well as heterogeneity in preferences for quality are required to generate nonzero demand for all the available cars.

As is well known, the assumptions of quasilinearity and no transaction costs imply that a consumer's optimal consumption decision in any period does not depend on her past and future choices, since her decisions are independent of income. Then, it is easy to verify that consumer θ's optimal car choice in period t is determined by simply comparing the *utility gains*

$$UG_t^k(\theta) \equiv \alpha_k \theta - p_t^k + \delta p_{t+1}^{v(k)} \tag{3}$$

across all choices $k = 1, \ldots, K+1$. Each utility gain is just the difference of $\alpha_k \theta$, the per-period flow of services that consumer θ obtains from car k, and $(p_t^k - \delta p_{t+1}^{v(k)})$, the implicit rental price

[12]This assumption is natural along the (pure strategy) equilibrium path of the dynamic model in the absence of shocks. When shocks are present, however, perfect foresight is no longer a palatable assumption, but the derivations in this model still obtain under the weaker assumption of rational expectations for the case where $F(\theta)$ is uniform. See footnote 27 for a more detailed exposition.

[13]Her discounted lifetime utility is deterministic for any sequence of cars that she chooses. This arises from the perfect foresight assumption.

[14]Since we normalize α_{K+1} to zero, we set $p_t^{K+1} = 0$, $\forall t$, accordingly. This implies that a car that dies in period t has zero resale value. In our empirical application, we adopt an alternative assumption that when cars die, consumers receive some positive, but exogenously given, scrappage value. In all cases, however, the scrappage sector is not endogeneized. This extension is described fully in Appendix Section B.1.

[15]See Prescott and Visscher (1977) for a static differentiated-product oligopoly model, and Bresnahan (1981) for its application to the automobile industry. While our scalar heterogeneity specification echoes the demand specification in these papers, much of the more recent empirical literature on the car industry (Berry, Levinsohn, and Pakes (1995), for example) has focused on modeling heterogeneity in consumer preferences by allowing for multiple dimensions of consumer heterogeneity. However, there are difficult conceptual as well as computational issues involved in extending models with multi-dimensional heterogeneity in preferences to a dynamic equilibrium framework. See Berry and Pakes (1999) for some discussion of the challenges in computing and estimating multi-dimensional vertical differentiation models.

paid for those services, where $\delta p_{t+1}^{v(k)}$ is the discounted resale price in tomorrow's secondary market. In every period, therefore, the optimal car choice rule dictates that consumer θ choose the option $k = 1, \ldots, K+1$ which offers the maximal utility gain given in equation (3).[16] We provide a formal proof of this result in Appendix A.

2.1.2 Deriving the demand functions

Given prices p_t^k, $p_{t+1}^{v(k)}$ for $k = 1, \ldots, K$ and quality levels $\alpha_1, \ldots, \alpha_K$, and subject to regularity conditions ensuring that each car model has positive demand (fully discussed at the end of this section), we derive the period t demand functions as follows. We find K cutoff values, $\tilde{\theta}_t^1, \ldots, \tilde{\theta}_t^K$, such that

$$\bar{\theta} \geq \tilde{\theta}_t^1 \geq \tilde{\theta}_t^2 \geq \tilde{\theta}_t^3 \geq \ldots \geq \tilde{\theta}_t^K \geq 0, \tag{4}$$

and all consumers with preference parameter $\theta \in [\tilde{\theta}_t^1, \bar{\theta}]$ consume car 1, all consumers with preference parameter $\theta \in [\tilde{\theta}_t^2, \tilde{\theta}_t^1]$ consume car 2, etc. Finally, all consumers with preference parameter $\theta \in [0, \tilde{\theta}_t^K]$ do not consume a car. Hence, $\tilde{\theta}_t^k$, for $k = 1, \ldots, K-1$, denotes the consumer who is indifferent between consuming cars k and $k+1$, and the last cutoff value, $\tilde{\theta}_t^K$, denotes the consumer who is indifferent between consuming the lowest-quality car K, and not consuming a car at all. Hence, these cutoff values solve the indifference conditions

$$\begin{aligned} \alpha_k \tilde{\theta}_t^k - p_t^k + \delta p_{t+1}^{v(k)} &= \alpha_{k+1} \tilde{\theta}_t^k - p_t^{k+1} + \delta p_{t+1}^{v(k+1)}, && \text{for } k = 1, \ldots, K-1, \\ \alpha_K \tilde{\theta}_t^k - p_t^k &= 0, && \text{for } k = K. \end{aligned} \tag{5}$$

Let x_t^k denote the demand for car k in period t, which is the (F-)measure of consumers for whom $UG_t^k(\theta) = \max_{k'} UG_t^{k'}(\theta)$, which is

$$x_t^k = \begin{cases} M(1 - F(\tilde{\theta}_t^k)), & \text{for } k = 1, \\ M(F(\tilde{\theta}_t^{k-1}) - F(\tilde{\theta}_t^k)), & \text{for } k = 2, \ldots, K. \end{cases} \tag{6}$$

Substituting these demand functions recursively in equation (6), we write the K cutoff values as

$$\tilde{\theta}_t^k = F^{-1}\left(1 - \frac{1}{M} \sum_{r=1}^{k} x_t^r\right), \quad \text{for } k = 1, \ldots, K. \tag{7}$$

Subsequently, substituting these expressions for the cutoff values into the indifference conditions given in equation (5), and imposing a natural nonnegativity constraint on market prices, we obtain

[16] This simplification of the consumers' problem relies on the assumption of no transaction costs: if (for example) we allowed for transaction costs in selling a used car, then a consumer's utility gain from choosing car $v(k)$ in period t would depend on whether she was endowed with car $v(k)$ at the beginning of period t (that is, whether she chose car k in period $t-1$). As a result, the consumer's utility maximization problem would be state dependent. See Anderson and Ginsburgh (1994), Porter and Sattler (1999) and Stolyarov (2000) for analyses of a durable goods market with transactions costs.

the inverse demand functions for each of the cars sold. In particular, a new car model $i \in \mathcal{L}$ with history $\eta(i), \upsilon(\eta(i)), \ldots, \upsilon^{T_i}(\eta(i))$ has inverse demand function

$$p_t^{\eta(i)} = \max\left\{ \left(\alpha_{\eta(i)} - \alpha_{\eta(i)+1}\right) F^{-1}\left(1 - \frac{1}{M}\sum_{r=1}^{\eta(i)} x_t^r\right) + \delta p_{t+1}^{\upsilon(\eta(i))} + p_t^{\eta(i)+1} - \delta p_{t+1}^{\upsilon(\eta(i)+1)}, 0 \right\}, \quad (8)$$

where the prices $p_{t+1}^{\upsilon(\eta(i))}, p_t^{\eta(i)+1}$ and $p_{t+1}^{\upsilon(\eta(i)+1)}$ are derived in analogous fashion.

The nonnegativity constraint on the inverse demand functions given by equation (8) implies that the primary and secondary markets can be in excess supply, in which case the constraint binds and the price equals zero. In order to maintain the tractability of the model, however, we assume that the nonnegativity constraints never bind in any market.[17] Under this assumption, we substitute prices recursively into equation (8) and obtain the inverse demand function for new model $i \in \mathcal{L}$ as

$$p_t^{\eta(i)} = \sum_{k=\eta(i)}^{K} \left(\alpha_k - \alpha_{k+1}\right) F^{-1}\left(1 - \frac{1}{M}\sum_{r=1}^{k} x_t^r\right) + \sum_{h=1}^{T_i-1} \delta^h \left(\sum_{k=\upsilon^h(\eta(i))}^{K} \left(\alpha_k - \alpha_{k+1}\right) F^{-1}\left(1 - \frac{1}{M}\sum_{r=1}^{k} x_{t+h}^r\right)\right). \quad (9)$$

We end this section with a brief consideration of the regularity conditions on prices p_t^k, $p_{t+1}^{\upsilon(k)}$ for $k = 1, \ldots, K$, and qualities $\alpha_1, \ldots, \alpha_K$ which are required to obtain period t demand functions in the cutoff manner described above. Essentially, these regularity conditions ensure that the cutoff values $\tilde{\theta}_t^1, \ldots, \tilde{\theta}_t^K$ are non-increasing in the quality of the good k, so that the demand for each car is positive. This implies that, for every car $k = 2, \ldots, K$, and every period t, the following inequality must be satisfied

$$\begin{aligned} &\frac{(p_t^{k-1} - \delta p_{t+1}^{\upsilon(k-1)}) - (p_t^k - \delta p_{t+1}^{\upsilon(k)})}{\alpha_{k-1} - \alpha_k} \\ \geq &\frac{(p_t^k - \delta p_{t+1}^{\upsilon(k)}) - (p_t^{k+1} - \delta p_{t+1}^{\upsilon(k+1)})}{\alpha_k - \alpha_{k+1}} \\ \geq &\frac{(p_t^{k+1} - \delta p_{t+1}^{\upsilon(k+1)}) - (p_t^{k+2} - \delta p_{t+1}^{\upsilon(k+2)})}{\alpha_{k+1} - \alpha_{k+2}} \\ \geq &0. \end{aligned} \quad (10)$$

A necessary (but not sufficient) condition for these inequalities to be satisfied is that the implicit rental prices follow the same order as their positions in the quality ladder quality ladder, i.e., for a given quality ladder $\alpha_1 \geq \cdots \geq \alpha_K$, the implicit rental prices are ordered

$$p_t^1 - \delta p_{t+1}^{\upsilon(1)} \geq p_t^2 - \delta p_{t+1}^{\upsilon(2)} \geq \cdots \geq p_t^K - \delta p_{t+1}^{\upsilon(K)}, \quad (11)$$

[17] For some parameter values, this assumption rules out the possibility that, in equilibrium, firms overproduce strategically in order to send some markets into excess supply. Esteban (2001) studies this issue for the case of a monopolist producer and shows that, in equilibrium, the secondary market will never be in excess supply for any depreciation of cars. For more general vertical differentiation structures, however, the result does not necessarily generalize.

for all periods t. As we discuss later, the requirement that these inequalities hold in every period t will raise issues for the empirical implementation.

2.2 The producers' dynamic problem

Having derived the inverse-demand functions for each car transacted, we now turn to the supply side of our model. The assumption that no market will be in excess supply (i.e., that the nonnegativity constraint in equation (8) not be binding) implies that the quantity demanded will equal the quantity supplied in all markets. Consequently, for each car model $i \in \mathcal{L}$, volumes in the secondary market evolve according to[18]

$$x_{t+h-1}^{\upsilon^{h-1}(\eta(i))} \equiv x_t^{\eta(i)}, \quad \text{for } h = 2, \dots, T_i. \tag{12}$$

For the rest of this section, we discuss the dynamic optimization problem faced by producers. We begin by deriving equations of motion for the cars which are actively traded.

First, we let $\boldsymbol{y}_t$ denote the vector of all cars-in-use (both new and used) in period t, defined as

$$\boldsymbol{y}_t \equiv [1, x_t^1, \dots, x_t^K]', \tag{13}$$

where the positions in this vector follow the ordering in the quality ladder.[19] Second, we define $\boldsymbol{d}_t$ as the L-dimensional vector of all new cars ($L \times 1$) produced in period t as

$$\boldsymbol{d}_t \equiv \left[x_t^{\eta(1)}, x_t^{\eta(2)}, \dots, x_t^{\eta(L)}\right]',$$

where the ordering of the models in $\boldsymbol{d}_t$ differs from the ordering in the quality ladder.

Given these definitions, we express the law of motion of the cars-in-use vector $\boldsymbol{y}_t$ as

$$\boldsymbol{y}_t = \boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{B}\boldsymbol{d}_t, \tag{14}$$

where $\boldsymbol{B}$ and $\boldsymbol{A}$ are matrices which, respectively, place new car models in the quality ladder and shift cars within the quality ladder as they age. Specifically, $\boldsymbol{B}$ is a $(K+1) \times L$ matrix with entries

$$\boldsymbol{B}(k+1, i) \equiv \begin{cases} 1, & \text{if } \eta(i) = k \\ 0, & \text{otherwise} \end{cases}, \quad \text{for } i = 1, \dots, L, \; k = 1, \dots, K, \tag{15}$$

and $\boldsymbol{A}$ is a $(K+1) \times (K+1)$ matrix with entries[20]

$$\boldsymbol{A}(k'+1, k+1) \equiv \begin{cases} 1, & \text{if } k' = k = 0 \\ 1, & \text{if } \upsilon(k) = k' \\ 0, & \text{otherwise} \end{cases}, \quad \text{for } k, k' = 1, \dots, K. \tag{16}$$

[18]Implicit in this equation is that there is no exogenous decrease in car stocks due to accidents. However, our model easily accommodates these decreases in stocks, so that $x_{t+h-1}^{\upsilon^h(\eta(i))} \equiv \zeta_{i,h} x_t^{\eta(i)}$, where the probability of an accident is $\zeta_{i,h} \in [0,1]$.

[19]As is standard in the matrix form formulation of linear-quadratic problems (and anticipating the linear-quadratic specification of this model), we set the first entry of $\boldsymbol{y}_t$ equal to 1 identically across all t.

[20]Note that the first row and column of $\boldsymbol{A}$ is filled with a one, to be consistent with the first entry in the $\boldsymbol{y}_t$ vector.

Next, let $C_j(x_t^{\eta(i)}; \forall i \in \mathcal{L}_j)$ denote the total cost of production for firm j. Then, we can write j's period t profit function as

$$\begin{aligned} \pi_t^j &= \sum_{i \in \mathcal{L}_j} p_t^{\eta(i)} \left(x_{t+\tau}^1, \dots, x_{t+\tau}^K; \tau = 0, \dots, T_i - 1 \right) x_t^{\eta(i)} - C_j(x_t^{\eta(i)}; \forall i \in \mathcal{L}_j) \\ &= \sum_{i \in \mathcal{L}_j} p_t^{\eta(i)} \left(\boldsymbol{y}_{t+\tau}; \tau = 0, \dots, T_i - 1 \right) x_t^{\eta(i)} - C_j(x_t^{\eta(i)}; \forall i \in \mathcal{L}_j) \\ &\equiv \sum_{i \in \mathcal{L}_j} \Pi^i \left(\boldsymbol{y}_{t+\tau}; \tau = 0, \dots, T_i - 1 \right), \end{aligned} \tag{17}$$

where $p_t^{\eta(i)}(x_{t+\tau}^1, \dots, x_{t+\tau}^K; \tau = 0, \dots, T_i - 1)$ denotes the inverse demand function in equation (9).

In this dynamic game, firms' production strategies at a given period t can become unwieldy because they can depend on the entire production history prior to period t. An appealing and natural assumption which overcomes this dimensionality issue is the Markov assumption that each firm's period t production is a function of only the past variables which affect current (period t) profits. These "payoff-relevant" state variables are $\boldsymbol{A y}_{t-1}$, the vector of the stock of cars produced prior to period t which are still actively traded in period t.[21] Hence, we focus on production strategies of the form

$$x_t^{\eta(i)} = g_i(\boldsymbol{A y}_{t-1}), \forall i \in \mathcal{L}_j, \ \forall j \in \mathcal{N}. \tag{18}$$

Then, for all periods, each firm $j \in \mathcal{N}$ chooses new car production $x_t^{\eta(i)}$, for all $i \in \mathcal{L}_j$, to maximize its period t-discounted profits anticipating the optimal future behavior of all producers of new cars by solving

$$\max_{x_t^{\eta(i)}, \forall i \in \mathcal{L}_j} \sum_{\tau=1}^{\infty} \sum_{i \in \mathcal{L}_j} \delta^{\tau-1} \left[\Pi^i \left(\boldsymbol{y}_{t+\tau}, \boldsymbol{y}_{t+\tau+1}, \dots, \boldsymbol{y}_{t+\tau+T_i-1} \right) \right], \tag{19}$$

subject to

$$\begin{array}{ll} \boldsymbol{y}_{t+\tau} & = \boldsymbol{A y}_{t+\tau-1} + \boldsymbol{B d}_{t+\tau} \\ x_{t+\tau}^{\eta(i)} & = g_i(\boldsymbol{A y}_{t+\tau-1}) \end{array}, \quad \text{for } \tau = 1, \dots, \infty. \tag{20}$$

Next, we write each firm's maximization problem as a dynamic programming problem with the state variable $A\boldsymbol{y}_{t-1}$. For this dynamic game, a **Markov perfect equilibrium** specifies decision rules, $g_i(\cdot)$, for all $i \in \mathcal{L}_j$, and $j \in \mathcal{N}$, and value functions $V_j(\cdot)$, and $j \in \mathcal{N}$, such that these solve the dynamic programming problems given by the Bellman equation

$$V_j(\boldsymbol{A y}_t) = \max_{x_t^{\eta(i)}, \forall i \in \mathcal{L}_j} \sum_{i \in \mathcal{L}_j} \Pi^i(\boldsymbol{y}_t, \boldsymbol{y}_{t+1}, \dots, \boldsymbol{y}_{t+T_i-1}) + \delta V_j(\boldsymbol{A y}_{t+1}), \tag{21}$$

[21] Note that the state vector is not $\boldsymbol{y}_{t-1}$, because this vector contains $x_{t-1}^{v^{T_i-1}(\eta(i))}$, the cars which have died between periods $t-1$ and period t, which cannot affect period t profits directly.

where

$$\boldsymbol{y}_{t+h} = \boldsymbol{A}\boldsymbol{y}_{t+h-1} + \boldsymbol{B}g(\boldsymbol{A}\boldsymbol{y}_{t+h-1}), \quad \text{for } h = 1, \ldots, T_i, \tag{22}$$

and

$$x_t^{\eta(i)} = g_i(\boldsymbol{A}\boldsymbol{y}_{t-1}), \quad \text{for } i \in \mathcal{L}_j. \tag{23}$$

The production strategies $g_i(\cdot)$, for all $i \in \mathcal{L}$, that solve this dynamic programming problem are *time consistent*, in the sense that firms correctly anticipate their own future optimal behavior.[22] Just as in the monopoly case, firms internalize the detrimental effects that current production would have on future secondary market prices, and therefore current (through the discounted resale price) and future profits, by curtailing current production. In our differentiated-product oligopoly, however, the magnitude of these effects also depends on the depreciation schedules of these cars: a firm producing cars which depreciate quickly faces less intense competition from its past production and, hence, tends to produce more. Analogously, a firm producing cars which depreciate slowly tends to produce less.

Furthermore, the position of one producer's cars, relative to those of its competitors, in the quality ladder, also affects its equilibrium production levels. Each producer indirectly benefits from increasing production if this decreases its rivals' future production. These benefits should be largest in magnitude for the high-end producers because, by producing more cars today, they intensify the future competition at the lower end of the market as these cars age. In our Cournot (strategic substitutes) setting, this can cause the low-end producers to reduce their production in the future. However, low-end producers cannot affect their high-end rivals in the same way, and the benefits they would derive from increasing production should be smaller accordingly. Moreover, since used cars are inferior to new cars, they will generally rank low in the quality ladder. Therefore, these used cars will especially affect demand for low-quality new cars, which can lead the producers of these cars to curtail production.

Although solving for the Markov-perfect equilibrium of our dynamic game can be computationally

[22] Time-consistency is equivalent to the principle of optimality for dynamic programming problems. Generally, firms can obtain a higher discounted profit stream by committing to future production paths $\left\{x_{t+\tau}^{\eta(i)},\ \forall i \in \mathcal{L}_j\right\}_{\tau=0}^{\infty}$ that maximize

$$\sum_{\tau=0}^{\infty} \sum_{i \in \mathcal{L}_j} \delta^t \pi_{t+\tau}^j \tag{24}$$

subject to the law-of-motion in equation (22). In this latter problem, we allow the firm to commit to (say) period $t+1$ production in period t, and therefore internalize the effect of $t+1$ production on period t profits. However, the solution to this problem is time inconsistent since, once period t passes, the firm no longer needs to internalize the effect of period $t+1$ production on her period t profits and, in the absence of commitment, would wish to change her period $t+1$ production plans. This commitment ability is eliminated in the former problem. Generally speaking, there are production paths which are feasible for the problem in equation (24), but cannot be recursively characterized via dynamic programming, and are therefore time-*inconsistent*. Hence, we can also refer to the optimal production paths from the problem in equation (24) and the dynamic programming problem in equation (21) as, respectively, the open and closed-loop problems.

severe,[23] we simplify its structure by specifying a linear-quadratic version of this model next.

2.3 Linear-quadratic specification

In order to derive the linear-quadratic formulation, we make three functional form assumptions. First, we assume that the preference parameter θ is uniformly distributed (with total measure M) over $[0, \bar{\theta}]$, so that $F(\theta) = \theta/\bar{\theta}$. This assumption implies that the inverse demand functions in equation (9) are linear and given by

$$\begin{aligned} p_t^{\eta(i)} =& \bar{\theta}\left(\alpha_{\eta(i)}\left(1-\sum_{r=1}^{\eta(i)}\frac{1}{M}x_t^r\right)-\sum_{r=\eta(i)+1}^{K}\alpha_r\frac{1}{M}x_t^r\right)+ \\ & \bar{\theta}\sum_{h=1}^{T_i-1}\delta^h\left(\alpha_{v^h(\eta(i))}\left(1-\sum_{r=1}^{v^h(\eta(i))}\frac{1}{M}x_{t+h}^r\right)-\sum_{r=v^h(\eta(i))+1}^{K}\alpha_r\frac{1}{M}x_{t+h}^r\right). \end{aligned} \tag{25}$$

Second, we assume the marginal costs of production are constant and independent across car models, so that $C_j(x_t^{\eta(i)}; \forall i \in \mathcal{L}_j) = \sum_{i\in\mathcal{L}_j} c_i$.[24] Finally, we assume that producers' production strategies $g_i(\cdot)$ are linear functions of the state and the value functions, $V_j(\cdot)$ are quadratic.[25]

By substituting these functional forms into the per-period profit function given by equation (17), we find that the firm's dynamic programming problem in equation (21) is a linear-quadratic problem in the state vector $\boldsymbol{A}\boldsymbol{y}_{t-1}$. That is, the value function $V_j(\cdot)$ is quadratic in the state, the firm's objective function in period t is quadratic in the control $x_t^{\eta(i)}$ (for $i \in \mathcal{L}$), and the optimal control function is linear in the state, Ay_{t-1}.

Next, we rewrite each firms' dynamic programming problem (given by equations (21)–(23)) in matrix notation. First, to rewrite the per-period profit function, we introduce K matrices, $\boldsymbol{R}_1, \ldots, \boldsymbol{R}_K$, which contain the linear coefficients from the inverse demand functions for cars $k = 1, \ldots, K$, respectively, in equation (25). Specifically, for each car model $i \in \mathcal{L}$, we introduce matrices $\boldsymbol{R}_{\omega(i,h)}$, for $h = 1, \ldots, T_i$, which are $(K+1)\times(K+1)$ matrices with zeros everywhere except for the $\eta(i)$-th column (the column that corresponds to the quality ranking of a new model i). This column is set to

$$[\alpha_{\eta(i)}\bar{\theta}, \underbrace{-\alpha_{\eta(i)}\frac{\bar{\theta}}{M}, \ldots, -\alpha_{\eta(i)}\frac{\bar{\theta}}{M}}_{\text{entries } 2,\ldots,\eta(i)}, \underbrace{-\alpha_{\eta(i)}\frac{\bar{\theta}}{M}, -\alpha_{\eta(i)+1}\frac{\bar{\theta}}{M}, \ldots, -\alpha_K\frac{\bar{\theta}}{M}}_{\text{entries } \eta(i)+1,\ldots,K+1}]'.$$

[23] See Ericson and Pakes (1995) and Pakes and McGuire (1994). Recently, Rust (1997) and Pakes and McGuire (2000) have developed stochastic algorithms to alleviate the "curse of dimensionality" in the computation of dynamic models with large state spaces (including dynamic oligopoly models).

[24] Quadratic cost functions $C_i(x_t^{\eta(i)}; \forall i \in \mathcal{L}_j) = \sum_{i\in\mathcal{L}_j}\left(c_{1i}x_t^{\eta(i)} + c_{2i}\left(x_t^{\eta(i)}\right)^2\right)$ can be easily accommodated in our framework. In our empirical work, we used both linear and quadratic costs functions, but focus on the results from the linear cost function specifications.

[25] By focusing on linear equilibrium production strategies, we abstract away from the collusive equilibria mentioned in footnote 4 above, which are implemented via nonlinear trigger (i.e., step-function) strategies.

Subsequently, each firm's Bellman equation (21) takes the matrix form

$$\boldsymbol{y}_{t-1}'\boldsymbol{A}'\boldsymbol{S}_j\boldsymbol{A}\boldsymbol{y}_{t-1} = \max_{x_t^{\eta(i)},\forall i\in\mathcal{L}_j}\left\{\sum_{i\in\mathcal{L}_j}\left[\sum_{h=1}^{T_i}\delta^{h-1}\boldsymbol{y}_{t+h-1}'\boldsymbol{R}_{\omega(i,h)}\boldsymbol{y}_t\right]\right\} - \boldsymbol{y}_t'\boldsymbol{C}_j\boldsymbol{y}_t + \delta\boldsymbol{y}_t'\boldsymbol{A}'\boldsymbol{S}_j\boldsymbol{A}\boldsymbol{y}_t, \quad (26)$$

where, for $h = 1, \ldots, T_i - 1$,

$$\boldsymbol{y}_{t+h} = \boldsymbol{A}\boldsymbol{y}_{t+h-1} + \boldsymbol{B}\boldsymbol{d}_{t+h}, \quad (27)$$

and

$$\boldsymbol{d}_{t+h} = \boldsymbol{G}\boldsymbol{A}\boldsymbol{y}_{t+h-1}. \quad (28)$$

In these equations, (i) $\boldsymbol{S}_j$ is the $(K+1)\times(K+1)$ matrix of coefficients in firm j's value function, which is quadratic in $\boldsymbol{A}\boldsymbol{y}_t$; (ii) $\boldsymbol{C}_j$ is the $(K+1)\times(K+1)$ cost matrix for firm j, which contains c_i in the $(1,\eta(i))$-th entry for all $i\in\mathcal{L}_j$ and zeroes everywhere else; and (iii) $\boldsymbol{G}$ contains the coefficients of the linear equilibrium production rule. In the rest of this section, we solve for the Markov-Perfect equilibrium of this problem and derive the $\boldsymbol{G}$ matrix, as a function of the underlying model parameters.

First, we substitute recursively the linear equilibrium production rule (equation (28)) into the law of motion for the cars transacted (equation (27)), and write the law of motion as

$$\boldsymbol{y}_{t+h} = \left[(\boldsymbol{I}+\boldsymbol{B}\boldsymbol{G})\,\boldsymbol{A}\right]^h\boldsymbol{y}_t, \text{ for } h = 1,\ldots,T_i - 1. \quad (29)$$

Then, substituting equation (29) into equation (26), we rewrite firm j's dynamic programming problem as

$$\begin{aligned}\boldsymbol{y}_{t-1}'\boldsymbol{A}'\boldsymbol{S}_j\boldsymbol{A}\boldsymbol{y}_{t-1} &= \max_{x_t^{\eta(i)},\forall i\in\mathcal{L}_j}\ \boldsymbol{y}_t'\left\{\left[\sum_{i\in\mathcal{L}_j}\sum_{h=1}^{T_i}(\boldsymbol{A}')^{h-1}[(\boldsymbol{I}+\boldsymbol{B}\boldsymbol{G})']^{h-1}\delta^{h-1}\boldsymbol{R}_{\omega(i,h)}\right] - \boldsymbol{C}_j + \delta\left[\boldsymbol{A}'\boldsymbol{S}_j\boldsymbol{A}\right]\right\}\boldsymbol{y}_t \\ &\equiv \max_{x_t^{\eta(i)},\forall i\in\mathcal{L}_j}\ \boldsymbol{y}_t'\boldsymbol{Q}_j\boldsymbol{y}_t.\end{aligned} \quad (30)$$

To solve for the equilibrium decision rule, we first let $\boldsymbol{B}_j$ denote the $(K+1)\times L_j$ matrix formed by extracting the columns of $\boldsymbol{B}$ corresponding to the $\mathcal{L}_j$ models produced by firm j. Also note that the quadratic form $\boldsymbol{y}'\boldsymbol{Q}\boldsymbol{y}$ is equivalent to $\frac{1}{2}\boldsymbol{y}'\left(\boldsymbol{Q}+\boldsymbol{Q}'\right)\boldsymbol{y}$. Therefore, the $L_j\times 1$-system of first order conditions for equation (30) becomes

$$\boldsymbol{B}_j'\left(\boldsymbol{Q}_j+\boldsymbol{Q}_j'\right)\boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{B}_j'\left(\boldsymbol{Q}_j+\boldsymbol{Q}_j'\right)\boldsymbol{B}\boldsymbol{d}_t = 0. \quad (31)$$

Define the $(L_j\times(K+1))$ matrices $\boldsymbol{W}_j \equiv \boldsymbol{B}_j'\left(\boldsymbol{Q}_j+\boldsymbol{Q}_j'\right)$ for each firm j, and the $(K+1)\times(K+1)$ matrix $\boldsymbol{W}\equiv[\boldsymbol{W}_1,\ldots,\boldsymbol{W}_N]'$. Then, stacking the systems of first-order conditions for all N firms as

$$\boldsymbol{W}\boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{W}\boldsymbol{B}\boldsymbol{d}_t = 0, \quad (32)$$

we write the industry-wide system of equilibrium decision rules as

$$\boldsymbol{d}_t = -(\boldsymbol{W}\boldsymbol{B})^{-1}(\boldsymbol{W}\boldsymbol{A})\,\boldsymbol{y}_{t-1}, \tag{33}$$

which take the form of the equilibrium decision rule given by equation (28), with

$$\boldsymbol{G} \equiv -(\boldsymbol{W}\boldsymbol{B})^{-1}\boldsymbol{W}. \tag{34}$$

In the present problem, we solve for the Markov-Perfect equilibrium production strategies using a value iteration procedure. We consider a long but finite-horizon version of the game and, starting from the terminal period, solve recursively for each firm's optimal production strategies via backwards induction. Under certain conditions, the sequence of production decision rules and value function coefficients converges to the unique linear Markov Perfect Equilibrium of the infinite-horizon game.[26] Consequently, for every set of parameter values, we use backward induction over the Bellman equation (30) to compute the equilibrium production strategies corresponding to those parameter values. Since the details of this procedure are standard, we leave them for the Appendix, Section D.4.[27]

3 Empirical Implementation

Given the generality of the model, it is difficult to gain a more precise understanding of equilibrium producer behavior without particular values for the model parameters. These parameters are (i) $\alpha_1, \ldots, \alpha_K$, the qualities of the competing cars; (ii) $c_1, \ldots, c_L$, the constant marginal production costs for each model; and (iii) $\bar{\theta}$, the upper bound of the consumer heterogeneity distribution.[28]

Rather than computing the model under different values of the parameters, we obtain values of these parameters by estimating a version of the linear-quadratic model using production and price data for the automobile industry, for the years 1971–1990. For new cars, we use data on list prices and quantities collected from past issues of *Ward's Automotive Yearbook*.[29] We manually compiled secondary market prices from back issues of the *Kelley Blue Book* (western US edition).[30] In the rest of this section, we describe our estimation procedure, as well as further assumptions we made in order to facilitate the empirical implementation of our model.

[26]See Başar and Olsder (1982), Section 5.5, for more details of these conditions for linear-quadratic games. We note that existence in the present oligopoly problem is not guaranteed since it depends on the parameter values; as in an oligopoly problem, we require that the reaction functions intersect over the relevant output range.

[27] In addition, a useful property of a linear-quadratic problem is the certainty equivalence property, which implies that the derivations in this section generalize if we introduce additive shocks to demand as well as production costs, as long as consumers and producers have rational expectations regarding future realizations of prices.

[28]As is usual in empirical dynamic models, the discount factor δ is not estimated, but rather fixed (at 0.95, in our case).

[29]This is the same dataset employed by Berry, Levinsohn, and Pakes (1999). It can be obtained from James Levinsohn's website, at `http://www.econ.lsa.umich.edu/~jamesl/verstuff/instructions.html`.

[30]We thank Bruce Hamilton for providing these old issues.

3.1 Estimation procedure

Although the important demand and supply relations in this market are given by linear equations (cf. equations (25) and (33)), least squares estimation of the reduced-form equations will not allow us to recover the structural parameters, since they are very nonlinear functions of the reduced-form regression coefficients.[31] Instead, we undertake direct structural estimation via a nested Generalized Method of Moments (GMM) procedure where a value iteration procedure to compute the equilibrium production rules is nested inside an outer loop which searches over parameter values matching the predicted population moments of the data-generating process (which are functions of the parameters) to their sample counterparts. In the rest of this section, we discuss the derivation of these moment conditions.

Up to this point, we have not introduced structural errors — factors observed by the agents in the model but unobserved by the econometrician — into the model.[32] Hence, the model does not generate as much randomness as we observe in the data. Specifically, the equilibrium ordering inequalities on prices and qualities of the cars (given in equation (10)) place restrictions on the values that the α parameters can take, given the observed prices. Since we assume these parameters to be constant over time (see next section), there is not enough variation in our model to explain the large variation in observed prices across time: essentially, without additional sources of randomness, our model is overidentified relative to the data. In order to overcome these difficulties, we assume that the observed prices and quantities are both perturbed by measurement error. This assumption is not difficult to justify, since the aggregation and averaging over sub-models required to obtain the prices and quantities for each car model potentially introduced some noise into the data.

Specifically, let $\hat{p}_t^k$ denote the observed price of car k, and $\hat{x}_t^{\eta(i)}$ denote the observed new production of a model i car, in period t. Let p_t^k and $x_t^{\eta(i)}$ denote their true counterparts (i.e., the quantities which would satisfy the demand and supply systems given by equations (25) and (33)). Then we assume that $\left(\hat{x}_t^{\eta(i)}, \hat{p}_t^k\right)$ differs from $\left(x_t^{\eta(i)}, p_t^k\right)$ via additive measurement error as follows

$$\begin{aligned} \hat{p}_t^k &= p_t^k + \epsilon_t^k, && \text{for } k = 1, \dots, K, \\ \hat{x}_t^{\eta(i)} &= x_t^{\eta(i)} + w_t^{\eta(i)}, && \text{for all } i \in \mathcal{L}. \end{aligned} \tag{35}$$

where ϵ_t^k and w_t^i are assumed to be (marginally) mean zero errors with arbitrary correlation across car models within period t, but independent across periods (and unbounded support).[33] In our

[31] Furthermore, reduced-form estimation is difficult due to the large number of reduced-form parameters. Note that the reduced-form for the supply-side alone (given by system of equations (33)) is a vector autoregression with $L \times (K+1)$ parameters (the elements of the $\boldsymbol{G}$ matrix). Estimating such a large number of parameters using our relatively short yearly time series on car production is infeasible.

[32] On the other hand, time-invariant model-specific effects (unobserved heterogeneity) are accommodated by estimating a separate α for each car model.

[33] This is similar to the approach taken in Bresnahan (1981); while Bresnahan assumes that the additive disturbances are normally distributed in order to estimate the parameters via maximum likelihood, we do not make explicit distributional assumptions, but rather assume population orthogonality conditions and estimate the parameters via GMM. These approach is also common in the equilibrium job search literature: for example, Eckstein and Wolpin (1990) and Ridder and van den Berg (1998) also rely on measurement error to generate randomness which makes the observed wages consistent with the equilibrium model.

discussion of the empirical results below, we calculate report the percentage of the inequalities in equation (10) which are satisfied at the estimated parameter values, for the observed prices. This gives some indication of the importance of measurement error in generating the price variation observed in the data.

We estimate the parameters of the model via GMM, using population moment restrictions that the measurement errors be mean independent of a vector of instruments. Since they are largely standard, we omit the details of the estimation procedure here, but present all the details in Appendix D.[34] Next we discuss specific assumptions employed in the empirical implementation of the model.

3.2 Assumptions for empirical implementation

Recall that $\boldsymbol{Ay}_{t-1}$, the state vector of our linear-quadratic model, has dimension $K+1$, where K is the number of different cars (including both new and used) transacted. Since $K = \sum_{i \in \mathcal{L}} T_i$, the dimensionality of the state vector grows very quickly in the assumed lifespan T_i of each model i. Hence, we curtail this "curse of dimensionality" by restricting the size of consumers' choice sets in several ways.

First, we assume that all cars (new or used) have constant quality over time. Therefore, the quality of a car depends only on its model and age, not on the year it was produced.[35] We assume that quality choice is exogenous and taken as given by all agents in this model (including car producers).

Second, we shorten the lifespan of all models to two years ($T_i = 2,\ \forall i \in \mathcal{L}$) and assume that one period in the theoretical model corresponds to one calendar year. We have performed simulations for a durable goods monopolist which show that production older than two or three years does not significantly affect current production decisions as long as the quality depreciation is not too small. Nevertheless, to minimize any potential distortions occasioned by this assumption on the demand side, we amend our model to award owners of one-year old cars an exogenous (but positive) residual payment for their cars. In this way, we capture the residual value of cars beyond their second year of life (see Appendix B.1 for more details). In all the specifications considered below, the residual payment for each model is set equal to the observed price of a two-year old version of the model, and assumed to vary over time.[36]

Third, we restrict our attention to seven models which we deem representative of the most popular automobile models during the twenty-year sample period considered (1971 to 1990). We consider the Ford Pinto and Escort, the Chevrolet Impala and Cavalier, the Oldsmobile Cutlass, the Toyota

[34] While we impose neither the inequalities in equations (10) nor (11) in estimating the parameters, we note that given the observed prices $\{\hat{p}_t^k, \forall k,\ \forall t\}$ and any quality ladder $\alpha_1, \ldots, \alpha_K$, a collection of measurement errors $\{\epsilon_t^k, \forall k,\ \forall t\}$ can be found such that the actual prices $\{p_t^k = \hat{p}_t^k - \epsilon_t^k, \forall k,\ \forall t\}$ satisfy the inequalities (10).

[35] The tractable linear-quadratic structure of the model would not prevail if we were to endogenize the quality/durability choice by each of the firms. However, as specification checks, we have re-estimated a version of the model allowing the quality ladder parameters (the α's) as well as the marginal cost parameters to be different in the 1980's versus the 1970's; since the results did not change qualitatively, we do not report them below.

[36] We also estimated specifications in which the scrappage value is fixed at an average value across all years; the results were largely robust to this specification check.

Camry, and the Honda Accord.[37] Most of these models either entered or exited the market during the sample period, and only the Oldsmobile Cutlass was available during the entire twenty-year period spanned by our data. Table 2 provides a summary of the market presence of these seven models. We assume that the entry and exit of models occurs exogenously, and is unforeseen by all agents.

We introduce two additional composite goods into consumers' choice sets to capture other automobile models available to consumers during the sample period. These two composites consist of (i) all other new cars; and (ii) all other one-year old used cars which do not correspond to one of the seven models listed above. Given these market definitions, the "outside good" for our empirical model is a composite product consisting of any used car two years old or older.[38]

We assume that the stocks of cars in these two composite categories evolve exogenously according to a random walk process and is taken as given by all agents (both producers as well as consumers). We provide details on how we accommodate this within our linear-quadratic structure in Section A.3 of the Appendix. Furthermore, to simplify the empirical implementation, these cars are assumed to last only one period.[39]

To summarize, the cars available to consumers during the sample period (1971–1990) are

- New Pinto and one-year old Pinto (only 1971–1980),
- New Escort and one-year old Escort (only 1981–1990),
- New Impala and one-year old Impala (only 1971–1985),
- New Cavalier and one-year old Cavalier (only 1981–1990),
- New Cutlass and one-year old Cutlass (all years),
- New Accord and one-year old Accord (only 1976–1990),
- New Camry and one-year old Camry (only 1983–1990),
- Any other new car (all years),
- Any other one-year old car (all years),
- Outside good: a used car with age exceeding one year (all years).

Additional details on the construction of the variables in our dataset are given in Appendix B.2. Table 1 provides summary statistics of these data. Given our assumptions, we will use the term "used" as shorthand for "one-year old" in what follows.

[37]In our empirical work, we assume that all Chevrolet and Oldsmobile models are produced by a single firm, i.e., General Motors.

[38]In our empirical work, we also consider different definitions of market size.

Earlier (in Section 2.1), we set α_{K+1} and p_t^{K+1}, the quality and price of the outside good, to zero, across all t. When the outside good is defined as a used car with age exceeding one year, by normalizing p_t^{K+1} to zero for all t, we implicitly assume that the price of the cars in this category maintains a constant differential with the other cars in consumers' choice sets. This is not an unreasonable assumption, the more so due to the difficulty in obtaining data on the average prices of used cars by age.

[39]We could have alternatively assumed that "Other one-year-old cars" in any period t equals the stock of "Other new cars" in the period $t-1$. However, one problem with this approach is that the total of cars transacted would not aggregate up to a market size equaling the cars in use figures from Polk (as reported in Cohen and Greenspan (1996)), which we use in our analysis.

4 Empirical findings

Estimation results from three specifications of the model are presented in Table 3. The three models differ in their definition of the market size, and therefore the definition of the outside option. In Model I, the market is the total cars in use, so that the outside option is all used cars aged two year or older. For Model II, the market is the total of cars in use aged three years or younger, so that the outside option becomes all used cars aged between 2 and 3 years. Finally, in Model III the market is the total of cars in use aged two years or younger, so that the outside option is all two-year old used cars.[40] The markups corresponding to our estimated magnitudes of marginal cost are presented separately in Table 4. The markups are evaluated at the average (across-time) list price of a car model, in 1983$.

The main results appear robust across all three specifications. Furthermore, the figures in the penultimate row of Table 3 indicate that, for all three specifications, more than half of the inequalities required for demand with a cutoff structure to obtain (cf. the end of Section 2.1.2) are satisfied at the estimated parameter values. This suggests that we are not attributing an undue proportion of the price variation in the data to measurement error.

Given the similarity in results across the three specifications, we will focus on the Model II results in what follows. Generally, the marginal cost estimates are precise, but not the quality ladder parameters (the α's). While this is not wholly surprising, given the relatively short sample span (twenty years) of our data, it suggests some caution be used in interpreting the results.

First, we explore the quality ladder parameter estimates (the α's). For expositional ease, we group cars on the basis of their estimated α's into four quality tiers (where cars are grouped into a common tier if their estimated α's are close in magnitude). While the estimates of the α's change across the three specifications, tier membership (as given in the "Tier" column of Table 3) is constant. The top Tier A contains only the "Other new cars" composite and the new Impala. Tier B contains the new Pinto as well as both the new and used Cutlass. Tier C, the largest tier, contains the new Escort, used Impala, new Cavalier, and both the new and used Camry and Accord. The lowest tier, Tier D, consists of used Escorts, Cavaliers, Pintos, and the "Other one-year old cars" composite.

These rankings are consistent with general market perceptions of these models. Furthermore, the relative sizes of the tiers reflect a "clustering" of products at the lower-end of the quality spectrum: this is a natural feature in vertically-differentiated durable-goods markets with depreciation, since high-quality new products evolve into lower-quality used products as they age.

Our results also show that the Cutlass, Accord, and Camry depreciate very slowly (in the sense that the new and used variants of each model are in the same tier), which is consistent with general market perceptions of these cars.[41] In addition, Table 1 shows that, generally, models which we estimate to

[40] See Appendix C for more details on market size.

[41] Both the Camry and the Accord were upgraded substantially in the early 1990s, and we conjecture that more recent data should show that these cars are no longer "Tier C" cars.

depreciate faster had lower production: for example, production of the slow-depreciating Camry and Accord averaged only 170,985 and 233,187 units (respectively) per year, while the Escort and Cavalier — both of which depreciate more quickly — averaged 352,725 and 305,531 units, respectively. This is consistent with our dynamic oligopoly model, where producers of slow-depreciating cars face more intense competition from their past production, and therefore have a greater incentive to lower equilibrium production.

The marginal cost estimates (cf. Table 4) indicate that markups are highest for the Escort and the Cavalier, at 51.4% and 57.1%, respectively. It is not surprising that these models are also the ones which had the highest production levels out of the seven models in our dataset since, *ceteris paribus*, cars with (relatively) lower production costs will be produced more. Similarly, we estimate the lowest markups for the Impala and Cutlass (23.95% and 27.03%, respectively), both of which had more modest production levels on average, during the sample period. The high estimated markup (45.2%) and low observed output of the Camry (45.2%) contradicts this trend. In our dynamic model, however, this is consistent with the Camry's low output. As we argued in the previous paragraph, the Camry's slow depreciation implies that new Camrys substitute well with used Camrys. This provides strong incentives for Toyota to curtail production of the Camry.

Dynamic equilibrium production rules Table 5 contains the equilibrium production rules computed from equation (33) using the results from Model II. Three sets of decision rules are presented, corresponding to the market definitions in 1971, 1981, and 1990.

The generally negative coefficients on the used car stocks indicate that equilibrium production is *decreasing* in stocks in the secondary market: this is due to the strategic substitutes aspect of the oligopoly quantity-setting game we consider. In addition, the largest (in magnitude) coefficients are attached to cars which are the closest substitutes. For example, for the 1981 choice set, we see that Impala production is more responsive to the production of "Other new cars" (coefficient -0.2596), than to past Impala production (coefficient -0.0685). This reflects our result that the "Other new cars" composite is a closer substitute for the new Impala (both are Tier A cars) than a used Impala (which is in Tier C). On the other hand, production of the new Cutlass is quite sensitive to the stock of used Cutlasses (coefficient -0.2342), since we estimate the Cutlass to depreciate very slowly (both the new and used Cutlass are in Tier B).

Finally, the slope coefficients in the equilibrium production rules are generally larger in magnitude for the lower-end cars (such as the Escort and the Accord, both in Tier C) than for the higher-end cars (such as the Impala and the Cutlass). This is a consequence of the clustering of products at the lower end of the quality spectrum, pointed out previously, which implies that new lower-end cars face a larger number of close substitutes than higher-end new cars. This asymmetry between higher-end and lower-end firms has important implications for the results from the simulated counterfactuals, to which we turn next.

4.1 Counterfactual simulations

Finally, we use our parameter estimates to examine two issues which have been investigated in the existing theoretical literature: the value of commitment in durable goods markets, and the profitability of a planned obsolescence strategy. Table 6 contains results from simulations employing the results from model specification II, for the 1981 choice set, which consists of the Ford Escort, Chevy Cavalier and Impala, and the Honda Accord.[42] The top panel of this table contains the simulated values for (1) steady-state equilibrium production (in millions of units); and (2) the corresponding per-period profits (in billions of dollars). The other panels in this table contain results from two sets of counterfactual simulations.

For all four car models, the simulated steady-state production levels are about one order of magnitude larger than the observed production levels (as given in Table 1). We believe that this lack of fit is due to the large number of alternatives in the car market, and the small (in absolute terms) market shares of the seven car models we consider. For these reasons, we report the results from the simulated counterfactuals in percentage changes relative to the baseline results, rather than in levels.

Profitability of full-commitment The second panel contains results from industry simulations under the counterfactual assumption that firms are able to commit fully to all future production levels.[43] The results suggest that the ability to commit leaves firms better off: profits from the Escort, Impala, Cutlass, and Accord would increase by 144%, 49%, 5% and 519%, respectively, relative to the baseline results, which were computed assuming no commitment. These results support Coase's conjecture that the inability to commit erodes a firm's profits. In the monopoly setting, which has been the focus of most of the Coase Conjecture literature, these higher profits can only achieved by decreases in production. In a durable goods oligopoly, however, this is not necessarily true: while production of the Impala and Cutlass decreases by 13% and 29.5%, respectively, the production of the Escort and the Accord actually increases, by 21% and 71%.

Reductions in durability: profitability of planned obsolescence strategy In a second set of counterfactual experiments, we explore the profitability of a planned obsolescence strategy for oligopolistic producers by simulating the effects of unilateral reductions in the durability for each car model. In all these simulations, we assume that marginal costs are not affected by the reduction in durability.

Panels 3–7 of Table 6 show that steady-state profits of three out of the four models increase sub-

[42] We also ran the simulations for the 1971 and 1990 choice sets, but they are qualitatively very similar to the 1981 results.

[43] We report the steady-state values of the full-commitment equilibrium path. See Appendix E for details on methodology employed in these simulations.

It is well-known that these full-commitment production levels can be implemented in a leasing equilibrium in which firms are forced to supply all their used vehicles to the secondary market. Hendel and Lizzeri (1999b) point out, however, that in a leasing equilibrium firms can generally do better by scrapping used vehicles rather than selling them in the secondary market; in essence, such scrappage allows firms to control the supply, and therefore prices, in the secondary market.

stantially relative to their baseline levels when they become less durable (the percentage increases are 635%, 71%, and 584%, respectively, for the Escort, Impala, and Accord). Profits for the Cutlass, on the other hand, decreased by 8.36% as a result of a reduction in durability. This can be explained by the Cutlass's slow quality depreciation (both the new and used Cutlass are in Tier B), which creates a large investment motive for buying the Cutlass. However, this investment motive disappears once we reduce its durability and, indeed, these simulation results indicate that this is enough to decrease the steady-state profits, despite the fact that the production of the less-durable Cutlass has increased by 28%.

As with the full commitment counterfactuals, there are significant differences in these results between the higher-end and the lower-end cars. For the Escort and the Accord, decreased durability not only increases the steady state production (and profits) of that product, but also decreases the production (and profits) of its rivals (by more modest percentages): for example, shortening the lifespan of an Escort increases its steady-state profits by 635.1%, but reduces the profits of all the other cars. The opposite result obtains for the higher-end Impala and Cutlass: for these models, decreased durability tends to increase production and profits across all car models. For example, reducing the durability of an Impala increases not only its steady-state profits by 71.8%, but also the profits of the Escort and the Accord.

The reason for these differences is apparent: a used Cutlass or Impala is a close substitute for a new Escort or Accord, so that decreasing the durability of the Cutlass and Impala would have a strong positive effect on Escort and Accord production, in the quantity-setting game we consider. On the other hand, a used Escort or Accord is not very closely substitutable with a new Impala or Cutlass. For the the Impala and the Cutlass, therefore, the production-curtailing effects arising from the higher production of new Escorts and Accords dominate the production-increasing effect arising from the elimination of used Escorts and Accords.

This immediately suggests that higher-end car producers may benefit more from the durability of their products than lower-end producers. Indeed, there may be an incentive for producers of higher-end models to make their cars more durable in order to compete more effectively with lower-end cars. Producers of lower-end cars, however, have the opposite incentive because they compete vigorously with their past production. This asymmetry in the profitability of a planned obsolescence strategy between producers at different ends of the quality spectrum appears to be a unique feature of a vertically-differentiated durable goods oligopoly where products depreciate over time.[44] Perhaps it also explains the absence of durable cars at the lower-end, and nondurable cars at the higher-end, of the car market.

In addition, these findings illustrate some effects of durability specific to the oligopolistic setting. In any durable goods market, rational forward-looking firms will recognize that current production will reach the secondary market in the future and, by lowering prices in those markets, can erode future

[44] However, this feature may not arise in durable goods markets when products can appreciate over time, such as wine or art markets.

profits. A durable goods monopolist fully internalizes this effect by curtailing current production. In a durable goods oligopoly, however, each producer internalizes only the effect this has on his own future profits, but not the detrimental effect it has on its rival's future profits.[45] Indeed, as pointed out by Bulow (1986), each oligopolistic producer derives an indirect benefit from increases in current production if this causes its rivals to lower production in the future, as will happen in the quantity-setting game we consider. In equilibrium, therefore, it may choose to overproduce today if these benefits outweighs the costs of more vigorous competition from the secondary market tomorrow. The simulation results presented in this section suggest that these incentives are larger for producers of higher quality cars, since their used cars are close substitutes for the new cars produced by lower-quality rivals.

5 Conclusions

In this paper we develop a model of dynamic oligopoly to understand the intertemporal links which arise from durability of the product and its trade in secondary markets. The tractable linear-quadratic features of this model enable us to develop an empirical model which we use to obtain estimates of the model's structural parameters and derive each producer's equilibrium decision rule. We use our results to simulate the model under reduced durability of the products, and thereby quantify the effects of durability on the car manufacturer's production decisions. To our knowledge, we are among the first to estimate such a dynamic model for the car industry.

The linear-quadratic structure has allowed us to uncover new insights into the effects of durability and secondary markets in the automobile industry, and in future work we would like to explore how these insights generalize in more flexible economic environments. We would like to extend the existing empirical model to accommodate transactions costs, as well as a richer parameterization of consumer heterogeneity. However, relaxing these assumptions leads to substantial difficulties, because the resulting model would no longer have a linear-quadratic structure. The corresponding challenges (both theoretical as well as computational) which arise will be tackled in future work.

[45] See Carlton and Gertner (1989) for an articulation of these ideas, in the context of mergers among durable goods producers, and Esteban (2001) for an analysis of how this effect influences the equilibrium dynamics of prices and production.

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A Dynamic optimality of the consumers' problem

Here we prove that our assumptions of quasilinear per-period utility and zero transactions costs imply that consumers' optimal car choices take the form described in Section 2.1.1. We consider a consumer of type θ who enters period t owning a used car of quality k (where $k = K+1$ denotes ownership of no car). Let $s_t \in \{1, \dots, K, K+1\}$ denote the car owned by consumer θ at the beginning of period t.

$W_t^\theta(s_t)$, consumer θ's value function at the beginning of period t, must satisfy the Bellman equation

$$\begin{aligned} W_t^\theta(k) &= \max\left\{\alpha_1\theta - p_t^1 + p_t^k + \delta W_{t+1}^\theta(v(1)), \dots, \alpha_K\theta - p_t^K + p_t^k + \delta W_{t+1}^\theta(v(K)), 0 + p_t^k + W_{t+1}^\theta(K+1)\right\} \\ &= p_t^k + \max\left\{\alpha_1\theta - p_t^1 + \delta W_{t+1}^\theta(v(1)), \dots, \alpha_K\theta - p_t^K + \delta W_{t+1}^\theta(v(K)), 0 + W_{t+1}^\theta(K+1)\right\} \\ &= p_t^k + W_t^\theta(K+1). \end{aligned} \tag{36}$$

Since this holds for all t, it implies that

$$W_{t+1}^\theta(k) = p_{t+1}^k + W_{t+1}^\theta(K+1), \tag{37}$$

Next, we substitute equation (37) into the second equation in (36) for each good k, and eliminate the constant terms common to all the utility levels (these are $W_{t+1}^\theta(K+1)$ and p_t^k). Therefore, the consumer determines her optimal consumption decision by solving

$$\begin{aligned} &\max\{\alpha_1\theta - p_t^1 + \delta p_{t+1}^{v(1)}, \dots, \alpha_K\theta - p_t^K + \delta p_{t+1}^{v(K)}, 0\} \\ =&\max\{UG_t^1(\theta), \dots, UG_t^K(\theta), 0\}, \end{aligned}$$

as posited in Section 2.1.1.

B Model extensions

B.1 Accommodating exogenous scrappage

In the current problem, the dimensionality of the state space grows quickly as we increase the durability of each car. In order to minimize this curse of dimensionality in the empirical implementation, we shorten the durability of cars. As described in Section 3.2, we assume that each car lasts only two years. To reduce the distortions arising from this assumption, we assume that owners of two-year old cars, while not being able to trade them in secondary markets, are able to obtain some nonzero scrappage value (or residual payment) for their cars. In this section we extend the model described in the main text to accommodate this scrappage value.

More precisely, we assume that each car model $i \in \mathcal{L}$ is scrapped after s_i (where $s_i \leq T_i$) periods of life for a scrappage value S_{it}.[46] We assume that consumers derive no utility from the consumption of a car older than s_i years. Hence, all consumers will scrap their cars at this age, since by doing so they obtain additional income in the form of the scrappage value.[47]

[46] We do not endogenize the sector for car scrappage; that is, we take the scrappage date s_i as given. This approach differs from, for example, Rust (1985b), in which the scrappage market is endogenized by allowing consumers to optimally choose the date at which to scrap their cars.

[47] If, instead, we had assumed that cars could be either scrapped or consumed, the scrappage value would then effectively constitute a price floor in the secondary markets. If this price floor binds, the linear-quadratic structure of the dynamic programming problem would not obtain.

For a given model $i \in \mathcal{L}$, in period t, we accommodate scrappage by reducing the lifespan of the car to $T_i = s_i$, and set the period t scrappage value S_{it} of this model equal to the resale price of car $(i, s_i + 1)$ in period t: $S_{i,t} \equiv p_t^{v^{s_i+1}(\eta(i))}$. With these changes, the inverse demand function for a new car i is

$$p_t^{\eta(i)} = (\alpha_{\eta(i)} - \alpha_{\eta(i)+1})\bar{\theta}\left(1 - \frac{1}{M}\sum_{r=1}^{k} x_t^r\right) + \delta p_{t+1}^{v(\eta(i))} + p_t^{\eta(i)+1} - \delta p_{t+1}^{v(\eta(i)+1)}. \tag{38}$$

After some recursive substitution, this becomes

$$\begin{aligned} p_t^{\eta(i)} =& \bar{\theta}\left(\alpha_{\eta(i)}\left(1 - \sum_{r=1}^{\eta(i)} \frac{1}{M} x_t^r\right) - \sum_{r=\eta(i)+1}^{K} \alpha_r \frac{1}{M} x_t^r\right) + \\ & \bar{\theta}\sum_{h=1}^{s_i-1} \delta^h \left(\alpha_{v^h(\eta(i))}\left(1 - \sum_{r=1}^{v^h(\eta(i))} \frac{1}{M} x_{t+h}^r\right) - \sum_{r=v^h(\eta(i))+1}^{K} \alpha_r \frac{1}{M} x_{t+h}^r\right) + \delta^{s_i} S_{i,t+s_i}. \end{aligned} \tag{39}$$

The linear-quadratic model then obtains by simply adding $\delta^{s_i} S_{i,t+s_i}$ to the $(1, \eta(i))$-th entry of the matrix $\boldsymbol{R}_i$. All the other derivations follow.

B.2 Modeling cars for which production evolves exogenously

The linear-quadratic structure of this model is easily extended to accommodate models for which production evolves exogenously (i.e., is not set endogenously by an agent within the model). For expositional convenience, we will refer to these models as "imports" in this section, and refer to those models produced by agents within the model as "domestic" models.

Let $\mathcal{L}_m$ denote the set of all import models and $L_m \equiv |\mathcal{L}_m|$ their total number. For each model $i \in \mathcal{L}_m$, let $T_i < \infty$ denote the number of periods it lasts. Then, the set of all models transacted is $\bar{\mathcal{L}} \equiv \mathcal{L} \cup \mathcal{L}_m$, where $\mathcal{L}$ was previously defined to be the car models produced (now domestically produced), and $\bar{L} \equiv |\bar{\mathcal{L}}|$ is their total number. Hence, the total number of car models equals $\bar{K} = \sum_{i \in \bar{\mathcal{L}}} T_i$.

Recall that in the derivation of the model we labeled car models $i \in \mathcal{L}$ as $i = 1, \ldots, L$. Accordingly, we label domestic models by $i = 1, \ldots, L$, and import models by $i = L + 1, \ldots, L + L_m$. First, we expand the quality ladder to incorporate the extra import models.

Second, we redefine the law of motion to incorporate the placement of import models. That is, we define a matrix $\boldsymbol{D}$ that places import models into the quality ladder. We let the dimensions of this matrix be $(\bar{K} + 1) \times L_m$ and define each entry as follows: for all $i = L + 1, \ldots, L + L_m$ and $k = 1, \ldots, \bar{K}$

$$\boldsymbol{D}(k + 1, i) = 1, \quad \text{if } \eta(i) = k,$$

and equal to zero, otherwise. Then, we write the law-of-motion of all cars in use as

$$\boldsymbol{y}_t = \boldsymbol{A}\boldsymbol{y}_{t-1} + \boldsymbol{B}\boldsymbol{d}_t + \boldsymbol{D}\boldsymbol{x}_{mt},$$

where $\boldsymbol{x}_{mt}$ denotes the vector of imports. We now follow the discussion in the main text to derive the demand functions, and equilibrium production rules.

In our empirical implementation, the "Other new car" and "Other one-year old car" categories are modeled as imports, whose stocks evolve exogenously according to a random walk process.

C Data appendix

Here we list additional assumptions made in constructing the dataset used in estimating the model.

The market size M differs across the three different specifications we estimated, the results of which are reported in Table 3. The three models differ in their definition of the market size, and therefore the definition of the outside option. In Model I, the market is the total of cars in use, and the outside option is all used cars aged two years or older. For Model II, the market is the total of cars in use aged three years or younger, so that the outside option becomes all used cars aged between two and three years. Finally, in Model III the market is the total of cars in use aged two years or younger, and the outside option is all two-year old used cars. The evolution of these three definitions of M during the sample period are as follows[48]

Year	Market size M, in millions		
	Model I	Model II	Model III
1971	60.4	32.3	24.4
1975	95.2	35.9	25.8
1980	104.6	36.7	26.8
1985	114.7	32.1	24.8
1990	123.3	36.1	25.9

Note that while the total number of cars in use (cf. column 1) rose dramatically across the twenty year sample period, the numbers of cars in use younger than 3 years (column 2) and younger than 2 years (column 3) remained largely constant, implying that the median age of a car in use rose significantly over the sample period.

Prices were deflated using a 1983 dollar as the base. Across all years, prices for the "Other new cars" category were taken from the "average expenditure per new car" series collected by the Bureau of Economic Analysis (BEA) at the US Department of Commerce. Since we were unable to obtain prices for used cars by vintage, the price of the "Other one-year old cars" category was set equal to 80% of the price of the "Other new cars".[49]

In all three model specifications, the scrappage values for the Pinto, Escort, Impala, Cavalier, Cutlass, Accord, and Camry in each year are set equal to the price of a two-year old model in that year (for example, the scrappage value for a Camry in 1987 is set to the secondary market price of a 1985 model-year Camry, in 1987). The scrappage value of the composite "Other new cars" in each year is set equal to the price of the "Other one-year old cars" in that same year. The scrappage value of the "Other one-year old cars" category in each year is set to 80% of the "Other one-year old cars" price in that same year.

D Estimation procedure: details

Let $\boldsymbol{\psi}$ denote the structural parameters of the model, which are (i) $\alpha_1, \ldots, \alpha_K$, the qualities of the competing cars; (ii) $c_1, \ldots, c_L$, the constant marginal production costs for the new cars; and (iii) $\bar{\theta}$, the upper bound of the consumer heterogeneity distribution. Let $\boldsymbol{X}$ denote the full set of observed quantities $\left[x_t^1, \ldots, x_t^K\right]'$, $t = 1, \ldots, T$ in the dataset. Let $\boldsymbol{\alpha} \equiv [\alpha_1, \ldots, \alpha_K]'$ denote the quality

[48]The source of this data is Polk, who compiled the data from car registration records. We thank Darrel Cohen for providing this data; see Cohen and Greenspan (1996) for more details on these data.

[49]We also estimated using a rate of 75%, and the results did not change appreciably.

ladder, the parameters of which we want to estimate.

D.1 Supply side

For the supply side, the basic estimating equations are the optimal policy functions linking current production of new cars to stock of used cars in the market (equations (33)). Substituting the law of motion, given by equation (27) into the equilibrium decision rule given by equation (33), we obtain

$$\begin{aligned}\boldsymbol{d}_t &= \boldsymbol{GAy}_{t-1} \\ &= \boldsymbol{GA}\left[\boldsymbol{A}^{t-1}\boldsymbol{y}_0 + \sum_{t'=1}^{t-1}\boldsymbol{A}^{t-1-t'}\boldsymbol{Bd}_{t'}\right],\end{aligned}$$

where $\boldsymbol{y}_0$ denotes the initial vector of cars in operation, in "year zero" (i.e., 1970 for our purposes). For convenience, we assume that $\boldsymbol{y}_0$ is measured without error; it is straightforward to incorporate measurement error in $\boldsymbol{y}_0$ also. Let $\hat{\boldsymbol{d}}_t \equiv \left[\hat{x}_t^{\eta(1)}, \ldots, \hat{x}_t^{\eta(L)}\right]'$, the vector of observed production in period t.

Moreover,

$$\begin{aligned}\hat{\boldsymbol{d}}_t &= \boldsymbol{GA}\left[\boldsymbol{A}^{t-1}\boldsymbol{y}_0 + \sum_{t'=1}^{t-1}\boldsymbol{A}^{t-1-t'}\boldsymbol{Bd}_{t'}\right] + \boldsymbol{w}_t \\ &= \boldsymbol{GA}\left[\boldsymbol{A}^{t-1}\boldsymbol{y}_0 + \sum_{t'=1}^{t-1}\boldsymbol{A}^{t-1-t'}\boldsymbol{B}\left(\hat{\boldsymbol{d}}_t' - \boldsymbol{w}_{t'}\right)\right] + \boldsymbol{w}_t \\ &= \boldsymbol{GA}\left[\boldsymbol{A}^{t-1}\boldsymbol{y}_0 + \sum_{t'=1}^{t-1}\boldsymbol{A}^{t-1-t'}\boldsymbol{B}\hat{\boldsymbol{d}}_{t'}\right] - \boldsymbol{GA}\left[\sum_{t'=1}^{t-1}\boldsymbol{A}^{t-1-t'}\boldsymbol{w}_{t'}\right] + \boldsymbol{w}_t \\ &\equiv \boldsymbol{GA}\hat{\boldsymbol{y}}_{t-1} + \boldsymbol{u}_t,\end{aligned}$$

where, given our assumptions, the error vectors $\boldsymbol{u}_t$ are correlated over time. Furthermore, the lagged structure of the decision rule in equation (33) implies that, given the assumed exogeneity of the $w_t^{\eta(i)}$'s, the composite error $u_t^{\eta(i)}$ will be correlated with the observed lagged output vector: i.e., $E(\boldsymbol{u}_t\hat{\boldsymbol{y}}_{t-1}) \neq 0$. For that reason, a regression of $\hat{\boldsymbol{d}}_t$ on $\boldsymbol{y}_{t-1}$ will not yield consistent estimates of the $\boldsymbol{G}$ matrix; furthermore, lagged output will not be a valid instrument, due to serial correlation in $\boldsymbol{u}_t$.

Therefore, we use lagged observed prices as instruments, since the measurement errors in the prices are assumed independent of the measurement errors in the quantities. Consequently, our supply-side population moment restrictions are

$$E(\boldsymbol{u}_t|\hat{\boldsymbol{p}}_{t-1}, \hat{\boldsymbol{p}}_{t-2}, \ldots) = 0, \tag{40}$$

where $\hat{\boldsymbol{p}}_t \equiv \left[\hat{p}_t^1, \ldots, \hat{p}_t^K\right]'$ denotes the instruments. The conditional moment restriction given by equation (40) implies that $\boldsymbol{u}_t$ is uncorrelated with any (vector) function $\boldsymbol{h}_1(\cdot)$ of the instruments: i.e., $E(\boldsymbol{u}_t * \boldsymbol{h}_1(\hat{\boldsymbol{p}}_{t-1}, \hat{\boldsymbol{p}}_{t-2}, \ldots)) = 0$. The sample analogue of these restrictions takes the form

$$\boldsymbol{\gamma}_T^s(\boldsymbol{\psi}) \equiv \frac{1}{T}\sum_t \left[\hat{\boldsymbol{d}}_t - (\boldsymbol{GA})\hat{\boldsymbol{y}}_{t-1}\right] * \boldsymbol{h}_1(\hat{\boldsymbol{p}}_{t-1}, \hat{\boldsymbol{p}}_{t-2}, \ldots). \tag{41}$$

The $\boldsymbol{G}$ matrix is, generally, a function of the model parameter $\boldsymbol{\psi}$. However, the mapping between $\boldsymbol{\psi}$ cannot be expressed analytically, since it depends on the coefficients of the equilibrium value function (cf. equation (33) in the text). Therefore, we use a nested procedure in order to construct the moment conditions for the supply side. First, for every parameter vector $\boldsymbol{\psi}$, we solve the linear quadratic dynamic programming problem given by equation (26) in order to obtain values for $\boldsymbol{G}(\boldsymbol{\psi})$, the matrix of production rule coefficients corresponding to the parameter vector $\boldsymbol{\psi}$. Second, we construct the sample moment conditions by plugging $\boldsymbol{G}(\boldsymbol{\psi})$ into equation (41).

D.2 Demand side

The estimating equation for the demand side are given by the inverse demand equations in (25). Note that the system of inverse demand functions in equation (25) is linear in the vector of quantities $\boldsymbol{X}$; let the matrix $\boldsymbol{\Phi}_t(\boldsymbol{\alpha})$ denote the matrix of coefficients (which are a function of the α's) for $\boldsymbol{p}_t \equiv \left[p_t^1, \ldots, p_t^K\right]'$, the vector of true period t prices.

Given this notation, we rewrite the system of inverse demand functions for period t as

$$\boldsymbol{p}_t = \boldsymbol{\Phi}(\boldsymbol{\alpha})' \cdot \boldsymbol{X}. \tag{42}$$

Let $\boldsymbol{w}$ denote the vector of measurement errors corresponding to the observed quantity vector $\hat{\boldsymbol{X}}$. Then,

$$\begin{aligned} \hat{\boldsymbol{p}}_t &= \boldsymbol{\Phi}_t(\boldsymbol{\alpha}) \cdot \left(\hat{\boldsymbol{X}} - \boldsymbol{w}\right) + \boldsymbol{\epsilon}_t \\ &= \boldsymbol{\Phi}_t(\boldsymbol{\alpha}) \cdot \hat{\boldsymbol{X}} - \boldsymbol{\Phi}_t(\boldsymbol{\alpha}) \cdot \boldsymbol{w} + \boldsymbol{\epsilon}_t \\ &\equiv \boldsymbol{\Phi}_t(\boldsymbol{\alpha}) \cdot \hat{\boldsymbol{X}} + \boldsymbol{v}_t. \end{aligned}$$

Given our independence assumptions on $\boldsymbol{\epsilon}_t$ and $\boldsymbol{w}_t$, the period t vectors of measurement error in prices and quantities, respectively, it is easy to derive that

$$E(\boldsymbol{v}_t | \boldsymbol{z}_t) = 0, \tag{43}$$

where $\boldsymbol{z}_t$ denotes a vector of instruments for period t. In our specifications, $\boldsymbol{z}_t$ consists of the constant 1 and $\hat{\boldsymbol{d}}_t$, as well as lagged quantities $\hat{\boldsymbol{d}}_{t-1}$, $\hat{\boldsymbol{d}}_{t-2}$, etc., as instruments.

As with the supply side in the previous section, the conditional moment restriction given by equation (43) implies that $\boldsymbol{v}_t$ is uncorrelated with any (vector) function $\boldsymbol{h}_2(\cdot)$ of the instruments: i.e., $E(\boldsymbol{u}_t * \boldsymbol{h}_2(\boldsymbol{z}_t)) = 0$. The sample analogue of these restrictions take the form

$$\gamma_T^x(\boldsymbol{\psi}) \equiv \frac{1}{T} \sum_t \left[\hat{\boldsymbol{p}}_t - \boldsymbol{\Phi}_t(\boldsymbol{\alpha}) \cdot \hat{\boldsymbol{X}}\right] * \boldsymbol{h}_2\left(\boldsymbol{z}_t\right).$$

D.3 GMM Estimation

Let $\boldsymbol{\mu}_T(\boldsymbol{\psi}) \equiv \begin{bmatrix} \gamma_T^s(\boldsymbol{\psi}) \\ \gamma_T^d(\boldsymbol{\psi}) \end{bmatrix}$. Our GMM estimator minimizes a quadratic form in $\boldsymbol{\mu}_T(\boldsymbol{\psi})$ given by

$$\boldsymbol{\mu}_T(\boldsymbol{\psi})' \boldsymbol{\Omega}_T^{-1} \boldsymbol{\mu}_T(\boldsymbol{\psi})$$

where $\boldsymbol{\Omega}_T$ is a (possibly deterministic) convergent sequence of weighting matrices.

Let ψ_T^{GMM} denote the vector of GMM estimates associated with a dataset with T periods of data. Under the usual assumptions, as $T \to \infty$ the sequence $\psi_T^{GMM} \xrightarrow{p} \psi_0$, and

$$\sqrt{T}\left(\psi_T^{GMM} - \psi_0\right) \xrightarrow{d} N\left(0, \left(\boldsymbol{J}_0\boldsymbol{\Omega}_0\boldsymbol{J}_0'\right)^{-1}\boldsymbol{J}_0\boldsymbol{\Omega}_0\boldsymbol{H}_0\boldsymbol{\Omega}_0\boldsymbol{J}_0'\left(\boldsymbol{J}_0\boldsymbol{\Omega}_0\boldsymbol{J}_0'\right)^{-1}\right), \tag{44}$$

where $\boldsymbol{\Omega}_0$ is the (probability) limit of the $\boldsymbol{\Omega}_T$ sequence,

$$\begin{aligned} \boldsymbol{J}_0 &= E_0 \frac{\partial \boldsymbol{\mu}(\psi)'}{\partial \psi}, \\ \boldsymbol{H}_0 &= \mathrm{Var}_0\left(\boldsymbol{\mu}(\psi)\right), \end{aligned}$$

and E_0 and Var_0 denote expectation and variance with respect to the true data-generating process (i.e., under ψ_0).

For the results reported in this paper, we employ a diagonal weighting matrix, in which each moment condition is weighted by the inverse of its (marginal) sample variance.

D.4 Deriving the equilibrium production rules

In this section we describe the value iteration procedure used to compute the Markov perfect equilibrium production rules. For all firms $j \in \mathcal{N}$, we begin with initial guesses for $\boldsymbol{S}_j^0,\ j \in \mathcal{N}$ for their respective matrices of value function coefficients. Using these matrices, we calculate recursively, for $\tau = 1, 2, 3, \ldots,$

$$\begin{aligned} \boldsymbol{Q}_j^\tau &\equiv \boldsymbol{A}'\boldsymbol{S}_j^\tau\boldsymbol{A},\ j \in \mathcal{N}, \\ \boldsymbol{W}_j^\tau &\equiv \boldsymbol{B}_j'\left(\boldsymbol{Q}_j^\tau + \boldsymbol{Q}_j^{\tau\prime}\right), \\ \boldsymbol{W}^\tau &\equiv \left[\boldsymbol{W}_1^\tau, \ldots, \boldsymbol{W}_N^\tau\right]', \\ \boldsymbol{G}^{\tau+1} &\equiv \left(\boldsymbol{W}^\tau\boldsymbol{B}\right)^{-1}\boldsymbol{W}^\tau. \end{aligned} \tag{45}$$

In each iteration, we update the coefficient matrix for the value functions via

$$\boldsymbol{S}_j^{\tau+1} = \left\{\left[\sum_{i\in\mathcal{L}_j}\sum_{h=1}^{T_i}(\boldsymbol{A}')^{h-1}[(\boldsymbol{I}+\boldsymbol{B}\boldsymbol{G}^{\tau+1})']^{h-1}\delta^{h-1}\boldsymbol{R}_{\omega(i,h)}\right] - \boldsymbol{C}_j + \delta\left[\boldsymbol{A}'\boldsymbol{S}_j^\tau\boldsymbol{A}\right]\right\}, \quad \text{for each } j \in \mathcal{N}.$$

We iterate this procedure until the sequence of matrices $\boldsymbol{S}_j^\tau$ and $\boldsymbol{G}^\tau$ converges. The converged values of these matrices are the coefficients of the equilibrium value functions and production rules, respectively.[50]

E Computing steady-state production levels under full commitment

In this section, we describe the procedure for simulating the steady-state full commitment production figures displayed in Table 6. Consider firm j, who produces all models in the set $\mathcal{L}_j$. Firm j's objective is to

$$\max_{\left\{x_t^{\eta(i)}\right\}_{t=1}^{\infty},\ i\in\mathcal{L}_j} \sum_{t=1}^{\infty}\pi_t^i. \tag{46}$$

[50] In practice, the value iteration process converged very quickly, often within 20 iterations.

Since each model lives for only two periods,

$$\pi_t^i = \boldsymbol{y}_t' \boldsymbol{R}_{\omega(i,1)} \boldsymbol{y}_t + \delta \boldsymbol{y}_{t+1}' \boldsymbol{R}_{\omega(i,2)} \boldsymbol{y}_t. \tag{47}$$

Consider firm j's optimal choice of $x_t^{\eta(i')}$, $i' \in \mathcal{L}_j$, for some future period t. Since each model lives for two periods only, $x_t^{\eta(i')}$ enters as an argument in π_{t-1}^i, π_t^i, and π_{t+1}^i, for all $i \in \mathcal{L}_j$.

So for each $i' \in \mathcal{L}_j$, production $x_t^{\eta(i')}$ satisfies the first-order condition

$$\delta^{-1} \sum_{i \in \mathcal{L}_j} \frac{\partial \pi_{t-1}^i}{\partial x_t^{\eta(i')}} + \sum_{i \in \mathcal{L}_j} \frac{\partial \pi_t^i}{\partial x_t^{\eta(i')}} + \delta \sum_{i \in \mathcal{L}_j} \frac{\partial \pi_{t+1}^i}{\partial x_t^{\eta(i')}} = 0, \tag{48}$$

where

$$\begin{aligned}
\frac{\partial \pi_{t-1}^i}{\partial x_t^{\eta(i')}} =& \delta \boldsymbol{B}_{i'}' \boldsymbol{R}_{\omega(i,2)} \boldsymbol{y}_{t-1}, \\
\frac{\partial \pi_t^i}{\partial x_t^{\eta(i)'}} =& \boldsymbol{B}_{i'}' \left(\boldsymbol{R}_{\omega(i,1)} + \boldsymbol{R}_{\omega(i,1)}' \right) \boldsymbol{y}_t + \delta \left(\boldsymbol{B}_{i'}' \boldsymbol{R}_{\omega(i,2)}' \boldsymbol{y}_{t+1} + \boldsymbol{B}_{i'}' \boldsymbol{A}' \boldsymbol{R}_{\omega(i,2)} \boldsymbol{y}_t \right), \\
\frac{\partial \pi_{t+1}^i}{\partial x_t^{\eta(i')}} =& \boldsymbol{B}_{i'}' \boldsymbol{A}' \left(\boldsymbol{R}_{\omega(i,1)} + \boldsymbol{R}_{\omega(i,1)}' \right) \boldsymbol{y}_{t+1} + \delta \boldsymbol{B}_{i'}' \boldsymbol{A}' \boldsymbol{R}_{\omega(i,2)} \boldsymbol{y}_{t+2},
\end{aligned} \tag{49}$$

and $B_{i'}$ is the column in matrix B corresponding to car model i'.

In steady-state, $\boldsymbol{y}_{t-1} = \boldsymbol{y}_t = \boldsymbol{y}_{t+1} = \boldsymbol{y}_{t+2} \equiv \boldsymbol{y}^*$. We substitute $\boldsymbol{y}^*$ into the expressions in equation (49) and solve the first-order condition (48) for the steady-state car stock vector $\boldsymbol{y}^*$. These steady-state production values are used to compute the percentage changes reported in the Section A of Table 6.

F Tables and Figures

Table 1: Data: summary statistics

Variable	T	Mean	Std Dev	Minimum	Maximum
Quantities					
Pinto (Ford)	10	282,764.50	114,052.90	142,467.00	480,472.00
Escort (Ford)	10	352,725.80	46,993.48	284,907.00	420,690.00
Impala (GM)	15	263,878.73	218,587.33	40,394.00	577,313.00
Cavalier (GM)	9	305,530.78	86,485.68	121,392.00	431,031.00
Cutlass (GM)	20	263,194.85	136,651.90	92,779.00	527,939.00
Accord (Honda)	15	233,186.80	116,126.72	18,643.00	417,179.00
Camry (Toyota)	8	170,985.38	79,750.45	52,666.00	284,595.00
Other new	20	5,229,327.85	924,296.26	3,438,773.00	6,684,767.00
Other one-year old	20	8,475,130.55	1,041,835.47	6,468,773.00	9,965,767.00
Prices (1983 dollars):					
New Pinto (Ford)	10	6,039.57	411.70	5,500.00	6,812.67
One-year old Pinto	10	5,688.91	309.98	5,263.16	6,253.44
New Escort (Ford)	10	6,599.60	566.32	5,477.38	7,126.61
One-year old Escort	8	5,638.76	157.96	5,422.86	5,848.54
New Impala (GM)	15	9,283.38	441.48	8,502.25	9,966.24
One-year old Impala	16	8,117.66	871.99	6,981.98	9,447.85
New Cavalier (GM)	9	8,368.76	547.03	7,880.65	9,589.66
One-year old Cavalier	8	7,202.44	227.80	7,007.43	7,642.49
New Cutlass (GM)	20	9,739.81	1327.74	7,855.86	11,897.48
One-year old Cutlass	20	8,730.03	937.16	6,747.57	9,984.70
New Accord (Honda)	14	9,790.65	999.33	7,927.51	11,276.42
One-year old Accord	13	9,309.72	645.51	8,140.81	10,025.10
Camry (Toyota)	8	9,797.89	1583.85	8,080.83	11,942.52
One-year old Camry	6	9,369.62	531.99	8,691.05	10,161.29
Other new	20	10,371.35	1201.10	9,044.62	12,127.64
Other one-year old	20	8,297.08	960.88	7,235.70	9,702.11

Table 2: Market history for new choice set

Year	Pinto	Escort	Impala	Cavalier	Cutlass	Accord	Camry
1971	1		1		1		
1972	1		1		1		
1973	1		1		1		
1974	1		1		1		
1975	1		1		1		
1976	1		1		1	1	
1977	1		1		1	1	
1978	1		1		1	1	
1979	1		1		1	1	
1980	1		1		1	1	
1981		1	1		1	1	
1982		1	1	1	1	1	
1983		1	1	1	1	1	1
1984		1	1	1	1	1	1
1985		1	1	1	1	1	1
1986		1		1	1	1	1
1987		1		1	1	1	1
1988		1		1	1	1	1
1989		1		1	1	1	1
1990		1		1	1	1	1

Table 3: Estimation results

Parameter	Model I Estimate	Model I Std. Error[a]	Model I Ladder Ranking	Model II Estimate	Model II Std. Error	Model II Ladder Ranking	Model III Estimate	Model III Std. Error	Model III Ladder Ranking	Tier
Quality ladder α's[b]										
New Pinto	1.0175	7.9034	4	1.0195	5.7395	4	1.0046	1.4674	4	B
One-year old Pinto	0.4655	5.6296	13	0.4369	4.0381	15(tie)	0.5284	1.1050	13	D
New Escort	0.7310	7.3193	9	0.7301	4.5521	9 (tie)	0.7435	1.4912	9	C
One-year old Escort	0.4441	5.8537	14(tie)	0.4441	3.5542	13	0.4465	0.8525	14(tie)	D
New Impala	1.2932	9.6966	2	1.2601	6.0237	2	1.0919	1.6685	2	A
One-year old Impala	0.6998	7.2354	12	0.7193	4.8027	11(tie)	0.7127	1.3102	12	C
New Cavalier	0.8593	8.1989	6	0.8182	4.7918	7	0.7983	1.4914	6	C
One-year old Cavalier	0.4441	5.6867	14(tie)	0.4439	3.2891	14	0.4465	0.7072	14(tie)	D
New Cutlass	1.0877	8.8811	3	1.0593	5.5225	3	1.0380	1.6243	3	B
One-year old Cutlass	1.0097	8.7432	5	1.0163	5.2365	5	1.0036	1.6716	5	B
New Accord	0.8593	7.9679	6	0.8305	4.8891	6	0.7591	1.4380	8	C
One-year old Accord	0.7257	7.3354	11	0.7193	4.6605	11(tie)	0.7133	1.3060	11	C
New Camry	0.7691	7.4787	8	0.7478	4.6072	8	0.7757	1.4503	7	C
One-year old Camry	0.7268	7.3472	10	0.7300	4.6993	9(tie)	0.7145	1.3145	10	C
Other new cars	1.3703	10.0307	1	1.3006	6.0733	1	1.1160	1.7057	1	A
Other one-year old cars	0.4424	5.7817	16	0.4369	3.4880	15(tie)	0.3999	1.0842	16	D
Marginal Costs ($'000):										
Pinto	5.0883	0.9290		3.9699	1.3053		6.0474	2.3808		
Escort	3.1698	0.6412		3.2462	1.3255		2.4667	6.2958		
Impala	7.5444	0.6937		7.0439	3.0760		6.6486	4.6558		
Cavalier	3.2300	0.4285		3.5642	1.5417		0.6630	3.3158		
Cutlass	7.3068	1.0364		7.1290	2.6870		8.3654	4.8672		
Accord	8.2000	0.2645		6.9636	1.6810		7.6456	4.7994		
Camry	6.1310	0.7454		5.4616	1.2034		6.8443	5.1261		
$\bar{\theta}$ ($'000)	4.6691	12.5401		5.3515	8.5207		22.0738	12.1227		
Market definition	All cars in use			All cars $\leq$ three-years old			All cars $\leq$ two-years old			
% of inequalities satisfied[c]	338/622 (54%)			338/622 (54%)			321/622 (52%)			
# moment restrictions	70			70			70			

[a] Using the asymptotic approximation $Var\hat{\theta} \approx \frac{1}{T}\Sigma_T$, where Σ_T is the finite-sample approximation of the variance-covariance matrix in equation (44).
[b] The α for the outside good has been normalized to 0.
[c] Percentage of the inequalities (equation (10) in the main text) which are satisfied, at the estimated parameter values.

Table 4: Estimated markups

Car Model	Avg. MSRP ($'000)	Model I	Model II	Model III
Pinto	6.0396	0.1575	0.3427	-0.0013
Escort	6.6788	0.5254	0.5140	0.6307
Impala	9.2626	0.1855	0.2395	0.2822
Cavalier	8.3163	0.6116	0.5714	0.9203
Cutlass	9.7702	0.2521	0.2703	0.1438
Accord	9.8667	0.1689	0.2942	0.2251
Camry	9.9681	0.3849	0.4521	0.3134

1971 Choice Set

$$\begin{bmatrix} x^{\text{Pinto}}_{new} & (B) \\ x^{\text{Impala}}_{new} & (A) \\ x^{\text{Cutlass}}_{new} & (B) \end{bmatrix} = \begin{bmatrix} 8.2902 & -0.2667 & -0.1971 & -0.2785 & -0.1197 & -0.2075 \\ 3.8369 & -0.2334 & -0.0251 & -0.0355 & -0.0153 & -0.1251 \\ 6.5740 & -0.1084 & -0.1484 & -0.2096 & -0.0901 & 0.0254 \end{bmatrix} \times \begin{bmatrix} 1 & \\ x^{\text{Other new}} & (A) \\ x^{\text{Impala}}_{one-year\ old} & (C) \\ x^{\text{Cutlass}}_{one-year\ old} & (B) \\ x^{\text{Pinto}}_{one-year\ old} & (D) \\ x^{\text{Other one-year old}} & (D) \end{bmatrix}$$

1981 Choice Set

$$\begin{bmatrix} x^{\text{Escort}}_{new} & (C) \\ x^{\text{Impala}}_{new} & (A) \\ x^{\text{Cutlass}}_{new} & (B) \\ x^{\text{Accord}}_{new} & (C) \end{bmatrix} = \begin{bmatrix} 6.0973 & -0.1300 & -0.2245 & -0.1595 & -0.2245 & -0.1386 & -0.2275 \\ 6.9488 & -0.2596 & -0.0685 & -0.1472 & -0.0685 & -0.0423 & -0.0551 \\ 7.8461 & -0.2342 & -0.1018 & -0.1930 & -0.1018 & -0.0628 & -0.0467 \\ 4.3974 & -0.1414 & -0.1544 & -0.1504 & -0.1544 & -0.0953 & -0.0998 \end{bmatrix} \times \begin{bmatrix} 1 & \\ x^{\text{Other new}} & (A) \\ x^{\text{Impala}}_{one-year\ old} & (C) \\ x^{\text{Cutlass}}_{one-year\ old} & (B) \\ x^{\text{Accord}}_{one-year\ old} & (C) \\ x^{\text{Escort}}_{one-year\ old} & (D) \\ x^{\text{Other one-year old}} & (D) \end{bmatrix}$$

1990 Choice Set

$$\begin{bmatrix} x^{\text{Escort}}_{new} & (C) \\ x^{\text{Cavalier}}_{new} & (C) \\ x^{\text{Cutlass}}_{new} & (B) \\ x^{\text{Accord}}_{new} & (C) \\ x^{\text{Camry}}_{new} & (C) \end{bmatrix} = \begin{bmatrix} 5.6100 & -0.1650 & -0.1539 & -0.1754 & -0.1083 & -0.1781 & -0.1083 & -0.1777 \\ 2.9293 & 0.1228 & 0.0051 & -0.0939 & -0.0580 & -0.0953 & -0.0580 & -0.2128 \\ 7.5746 & -0.3522 & -0.2147 & -0.0420 & -0.0259 & -0.0426 & -0.0259 & 0.0716 \\ 4.6216 & -0.1906 & -0.1627 & -0.1251 & -0.0772 & -0.1269 & -0.0772 & -0.0674 \\ 6.4671 & -0.1662 & -0.1339 & -0.1660 & -0.1025 & -0.1685 & -0.1025 & -0.0839 \end{bmatrix} \times \begin{bmatrix} 1 & \\ x^{\text{Other new}} & (A) \\ x^{\text{Cutlass}}_{used} & (B) \\ x^{\text{Accord}}_{one-year\ old} & (C) \\ x^{\text{Cavalier}}_{one-year\ old} & (D) \\ x^{\text{Camry}}_{one-year\ old} & (C) \\ x^{\text{Escort}}_{one-year\ old} & (D) \\ x^{\text{Other one-year old}} & (D) \end{bmatrix}$$

Table 5: Equilibrium decision rules
Using Model II Results
Tier in parentheses (see Table 3 for definition).

Table 6: Simulated Counterfactuals
Using Model II Results

New Escort (C)[a]	New Impala (A)	New Cutlass (B)	New Accord (C)
Baseline results: reported figures are *Output* (millions of units) *Single-period profits* ($billions)			
1.0953	1.9404	3.0448	0.5168
0.2241	0.9876	2.7160	0.0571
Counterfactuals: reported figures are *percentage changes* relative to above baseline results			
A. Full commitment results:			
20.83	-13.14	-29.50	71.30
144.07	48.92	5.19	519.43
B. Effects from reduced durability in:			
One-year old Escort (D)			
248.62	-11.95	-3.15	-86.94
635.11	-24.92	-9.74	-98.36
One-year old Impala (C)			
20.17	52.00	-1.03	35.99
43.05	71.76	-5.56	82.27
One-year old Cutlass (B)			
20.16	22.48	27.98	70.28
42.61	47.36	-8.36	185.76
One-year old Accord (C)			
-11.55	-5.79	-2.07	237.54
-22.69	-13.01	-7.78	584.86

[a]Tier, as defined in Table 3, in parentheses.