Increasing Competition and the Winner's Curse: Evidence from Procurement*

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Abstract

We empirically measure the effects of increasing competition on equilibrium bidding in procurement auctions. In common-value auctions, the winner's curse counsels more conservative bidding as the number of competitors increases. First, we estimate the structural parameters of an equilibrium bidding model and test for the importance of common-value components in bidders' preferences. Second, we use these estimates to calculate the effects of increasing competition on both individual bids as well as winning bids, i.e.,. procurement costs.

We analyze bid data from construction procurement auctions run by the New Jersey transportation department. Our results indicate that, for a large subset of these auctions, the median procurement cost *rises* as competition intensifies: increasing the number of bidders from 3 to 6 raises median procurement costs by about 15%. In this setting, then, asymmetric information overturns the common economic wisdom that more competition is always desirable.

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1 Introduction

How do increases in competition affect equilibrium bidding at auctions? According to the Walrasian analogy of markets as auctions, an increase in the number of bidders should encourage more aggressive bidding, so that in the limit, as the number of bidders becomes arbitrarily large, the imperfectly-competitive auction setting approaches the efficient perfectly-competitive outcome.

While this is true within the independent private-values paradigm, it may not be true at auctions with affiliated signals, and an important subset of these are common-value auctions, in which the competing bidders are differentially (but incompletely) informed about the value of the object which they are trying to win. A distinctive feature of common-value auctions is the *winner's curse*, an adverse-selection problem which arises because the winner tends to be the bidder with the most overly-optimistic information (or "signal") concerning the object's value. Bidding naïvely based simply on ones information would lead to negative expected profits so that, in equilibrium, a rational bidder internalizes the winner's curse by bidding less aggressively.[1]

In common-value settings, however, an increase in the number of bidders has two counteracting effects on equilibrium bidding behavior. First, the increased competition generally leads to more aggressive bidding, as each potential bidder tries to maintain her chances of winning against more rivals: we call this the *competitive effect*. Second, the winner's curse becomes more severe as the number of potential bidders increases, and rational bidders will bid less aggressively in response: we call this the *winner's curse effect*. If the winner's curse effect is large enough, the possibility arises that prices could actually rise as the number of competitors increases. Recently, Bulow and Klemperer (1999), Krishna and Morgan (1997), and Bordley and Harstad (1996) have pointed out this possibility; as these authors note, the winner's curse provides a prominent example where asymmetric information can overturn the common economic wisdom that more competition is always desirable. More recently, Pinkse and Tan (2000) (hereafter PT) have shown that this decrease in bidding aggressiveness as competition increases can occur generally in affiliated value first-price auctions, so that while the winner's curse does not arise in private-value settings, an *affiliation effect* in affiliated private-value models also leads to less aggressive equilibrium bidding as competition increases, similar to the effect of the winner's curse in common value auctions.[2]

In this paper, we investigate the empirical importance of these considerations using bid data from construction procurement auctions run by the New Jersey department of transportation in the years 1989–1997. We address two questions in turn. First, we examine the importance of common-value components in bidders' preferences. Second, provided we find some evidence of common values, we

[1] More precisely, in the procurement (low-bid) setting, even if any bidder's signal X is an unbiased estimate of the unknown (but common) project cost c (i.e., $E(X) = c$), conditional on winning the signal is a downwardly-biased estimate of w (i.e., $E(X|\text{win}) < c$). This implies that if a bidder were to naively bid her unbiased signal X, her expected profits $E(X|\text{win}) - c$ would be negative.

[2] While we focus on the effects of the winner's curse in this paper, we note that the PT result implies that winner's curse-like effects can arise even in private values settings, as long as signals are affiliated. One advantage of our parametric approach is that we can explicitly test whether any observed decrease in bidding aggressiveness as competition increases is due to winner's curse effects, or to PT's "affiliation effect".

investigate how the winner's curse affects equilibrium bidding. We do this by quantifying two comparative statics which have been the focus of the theoretical work on common-value auctions. First, does an increase in the number of competitors lead a given bidder to bid more or less aggressively, in equilibrium? Theoretical examples in Smiley (1979) and Matthews (1984) have shown that the result often depends on both the parametric assumptions on the information structure, as well as the specific values of the parameters, and we measure the effects using the parameter estimates for our model of equilibrium bidding. Second, and more important for policy purposes, does the winning bid rise or fall as competition increases? Previous theoretical work (e.g., Wilson (1977), Milgrom (1979)) have studied convergence of the winning bid in large CV auctions. In contrast, we measure these effects for the (often modest) range of bidders which we observe in real-world auctions.To our knowledge, we are the first to address these issues empirically.

The procurement setting is particularly pertinent for the issues raised above. Although exact figures are difficult to obtain, McAfee and McMillan (1987) (p. 3) state that "the national, regional, and local governments in a typical modern market economy together spend between one-quarter and one-third of national income on goods and services [...]; of this amount, perhaps one-half [...] is paid by governments to firms." Many of these payments are for contract work awarded via low-bid auctions identical to those considered here, so that there appear to be important efficiency and revenue lessons to be learned from the results.

Our results show that different types of contracts differ substantially in the degree that private and/or common value components are important. We find that both common-value and private-value components are important in auctions for highway-work and bridge-repair contracts, while more homogeneous road paving contracts are predominantly private-value auctions. These results indicate that the winner's curse is particularly strong in highway-work auctions: simulated bid functions show that the median bidder's equilibrium bid increases from about \$0.2 million with 2 bidders to above \$0.6 million with 10 bidders.[3] Moreover, winning bid simulations indicate that the average procurement cost is strictly increasing in the number of bidders as competition intensifies: for example, the average costs rise about 15%, as the number of bidders increases from 3 to 6. For these auctions, the optimal number of participants or potential bidders (which would minimize expected procurement costs) would be 3. Clearly, then, there are cases where the "law of demand" is violated: an increase in competition leads to *higher* procurement costs.

In the next section we describe the model of equilibrium bidding which we employ in our work. In section 3, we introduce our data on New Jersey transportation department procurement auctions, and discuss institutional features which affect our model specification, which we introduce in section 4. Section 5 contains the empirical results, and in section 6 we discuss the policy implications of our results. Section 7 concludes.

[3]Furthermore, simulations also show that these effects arise from the winner's curse effect, not from the affiliation effect described by PT.

2 Equilibrium bidding at low-price procurement auctions

In our empirical work, we employ a structural approach which allows us to recover bidders' equilibrium strategies. These are required for our investigation into how increasing competition affects equilibrium bidding behavior.[4] We build on the previous literature by considering a model where bidders' valuations have both private and common-value components. Such a model seems especially warranted for procurement settings, where uncertainty about future input prices could drive common-values but differences in input efficiency across firms could drive private-values.[5]

To our knowledge, this paper is among the first to tackle structural estimation of an equilibrium bidding model where bidders' valuation have both private and common value components. Previously, Laffont and Vuong (1996) argued that it is impossible to distinguish nonparametrically a common value model from a private value model. Subsequently, others have argued that while it is not possible to nonparametrically identify the joint latent distribution of the common value element and bidders' signals, it *is* possible to test nonparametrically between a common and private value paradigm. Hendricks, Pinkse, and Porter (1999) formulated nonparametric tests by exploiting the presence of a binding reserve price, and in ongoing work (Haile, Hong, and Shum (2000)), we propose tests which exploit exogenous variation in the number of bidders. In this paper, however, we adopt a parametric approach to recover the joint distribution of bidders' valuations and signals which characterizes the information structure.

Next, we briefly describe equilibrium bidding behavior in single-unit, low-price procurement auctions.[6] We delay discussion of the specification of contractors' costs until later.

An auction has n risk-neutral contractors (indexed $i = 1, \ldots, n$), each of whom has a cost c_i associated with completing the project, and receives a private signal x_i about c_i. Contractor i observes only x_i prior to the beginning of the auction, but not any of the costs, c_j, for $j = 1, \ldots, n$, or any of her rivals' signals, x_j, for $j \neq i$.

The contractors' costs and private signals are assumed to be distributed according to the distribution function $\tilde{F}(c_1, \ldots, c_n, x_1, \ldots, x_n; \theta)$ parameterized by the vector of parameters θ, which are

[4] The seminal empirical auction papers in the structural vein are by Paarsch (1992) and Laffont, Ossard, and Vuong (1995). Most recently, important progress has been made in the structural estimation of private-values auction models (see Bajari (1998); Guerre, Perrigne, and Vuong (2000); Li, Perrigne, and Vuong (2000); and Deltas and Chakraborty (1997)).

[5] In previous work (Hong and Shum (1997)), we have empirically implemented an equilibrium ascending-price auction model also allowing for both private and common values. Previously, Bajari (1998) and Jofre-Bonet and Pesendorfer (1999) have modeled procurement auctions in a private-values framework. Recently, Hendricks, Pinkse, and Porter (1999) and Bajari and Hortacsu (1999) have also considered common-value models recently, but they consider pure common value (i.e., "mineral rights") models of competitive bidding. Our model has both common and private values.

In a previous study of procurement auctions, Thiel (1988) concluded that observed bids do indeed reflect winner's curse considerations, but we take the analysis one step further by showing what this implies about equilibrium bids and, more important, equilibrium procurement costs.

[6] The assumption of rational equilibrium bidding which characterizes our approach is potentially at odds with findings in the experimental literature (cf. Kagel and Levin (1986)), which find that bidders only learn to internalize the winner's curse (i.e., bid "rationally") through experience. In the procurement setting, however, the bidders are by and large experienced firms, so we feel our assumption of rational bidding is justified.

the parameters of interest in the estimation process.[7] As we describe in the next section, we consider a specification which allows both common-value and private-value components in bidders' cost functions.[8]

The low-price auction proceeds as follows: observing x_i, contractor i chooses a bid b_i to maximize his expected payoff, given the other contractors' equilibrium behavior:

$$b_i = \text{argmax}_b \mathcal{E}_{x_j, j \neq i} \mathcal{E}_{c_i | x_1, \ldots, x_n} \left[(b - c_i) \mathbf{1} \left(x_j \geq s_{j,n}^{-1}(b), j \neq i \right) | x_i \right]$$

where $s_{i,n}(\cdot), i = 1, \ldots, n)$ denotes the equilibrium bidding strategy (or *bid function*) for contractor i in an n-bidder auction.

We assume that contractors are *symmetric*, in the sense that the joint distribution $\tilde{F}$ is exchangeable with respect to the indexes $1, \ldots, n$.[9] As is standard, we assume that the random variables $(C_1, \ldots, C_n, X_1, \ldots, X_n)$ are affiliated.[10] Given these assumptions, there is a unique pure-strategy Bayesian Nash equilibrium in which each contractor bids according to identical, monotonically increasing strategies; i.e., $s_j(\cdot) \equiv s(\cdot), \ \forall j$.[11]

The first-order condition of this maximization problem is (cf. (Milgrom and Weber, 1982, pg. 1107))

$$s_n(x_i) = \frac{s_n'(x_i)\left[1 - F_{-i}(x_i|x_i)\right]}{f_{-i}(x_i|x_i)} + v_n(x_i, x_i), \tag{1}$$

a differential equation which defines the equilibrium bidding strategy $s_n(x_i)$. In this equation, $v_n(\cdot, \cdot)$ is the conditional expectation

$$v_n(x, y) = \mathcal{E}\left[c_i | x_i = x, \min_{j \in [1,n]; j \neq i} x_j = y \right],$$

where the expectation is taken over the posterior distribution of C_i conditional on the joint event $\left(x_i = x, \min_{j \in [1,n]; j \neq i} x_j = y\right)$, and $f_{-i}(\cdot|\cdot)$ denotes the conditional density of $\min_{j \neq i} x_j$ conditional on x_i. After integrating out this differential equation, the equilibrium bid function can be expressed as (cf. Milgrom and Weber (1982) (theorem 14))

$$s_n(x_i) = v_n(x_i, x_i) + \int_{x_i}^{\bar{x}} \left\{ \exp\left[-\int_{x_i}^{\alpha} \frac{f_{-i}(s|s)}{1 - F_{-i}(s|s)} ds \right] \right\} v'(\alpha, \alpha) d\alpha \tag{2}$$

[7] We delay discussion of our parametric assumptions until section 4 below.

[8] In the pure *private-value* paradigm, $C_i = X_i \ \forall \ i$ (i.e. each bidder knows his true valuation for the object) while in the pure *common-value* paradigm $C_i = C, \ \forall \ i$ (i.e. the value of the object is the same to all bidders, but none of the bidders knows the true value of the object; here the individual X_i's are noisy signals of the true but unknown C).

[9] Milgrom and Weber (1982) provided the seminal analysis for the symmetric versions of most of the usual auction forms. However, much of the recent theoretical work has focused on asymmetric cases (Maskin and Riley (2000), Bulow, Huang, and Klemperer (1999), Lebrun (1999)). In ongoing research, we are considering an extension of our estimation methodology to asymmetric bidding environments, resembling Campo, Perrigne, and Vuong (1998). In the next section we discuss an explicit test of the symmetry assumption using our data.

[10] cf. Milgrom and Weber (1982). Affiliation roughly implies that large values for some of the variables make the other variables more likely to be large than small. Given two n-vectors $\mathbf{X}_1$ and $\mathbf{X}_2$ which are *i.i.d.* realizations from F, let $\bar{\mathbf{X}}$ denote the componentwise maximum of $\mathbf{X}_1$ and $\mathbf{X}_2$, and $\underline{\mathbf{X}}$ the component-wise minimum. Affiliation means that the likelihood of $(\bar{\mathbf{X}}, \underline{\mathbf{X}})$ is at least as high as that of $(\mathbf{X}_1, \mathbf{X}_2)$.

[11] Symmetry and monotonicity imply that $b_i > b_j \Leftrightarrow s_n(x_i) > s_n(x_j) \Leftrightarrow x_i > x_j$. Analogously, the event that bidder i wins can be simplified: $\mathbf{1}\left(x_j \geq s_{j,n}^{-1}(b_i), j \neq i\right) = \mathbf{1}\left(x_j \geq s_n^{-1}(b_i), j \neq i\right) = \mathbf{1}\left(\min_{j \neq i} x_j \geq x_i\right)$, since $x_i = s_n^{-1}(b_i)$ in equilibrium.

and the n subscript emphasizes that for a given x_i, the equilibrium bid $s_n(x_i)$ varies for different n.

2.1 Winner's curse, competitive and affiliation effects

The *competitive*, *affiliation* and *winner's curse* effects alluded to earlier are distinguishable in equation (1) above. From this equation we see that an equilibrium bid $b_i = s(x_i)$ is governed by two important components: (1) the $\frac{s'(x_i)[1-F_{-i}(x_i|x_i)]}{f_{-i}(x_i|x_i)}$ term and (2) the $v_n(x_i, x_i)$ term. Given our affiliation assumptions, an increase in n will increase the second term $v_n(x_i, x_i)$, holding x_i fixed (cf. (Milgrom, 1982, section 6)).[12] This is the winner's curse effect. On the other hand, an increase in n will generally have an ambiguous effect on the first term. On the one hand, the $[1 - F_{-i}(x_i|x_i)]$ portion of the numerator of the first term is essentially the probability of winning the low-bid auction for a given signal x_i, and this probability shrinks to zero as n increases. Thus for large n, the first term should be decreasing in n: we call this the *competitive effect*. However, PT note that in the affiliated private values framework, for some intermediate values of n and some signals x_i, the hazard rate $f/(1-F)$ could be increasing in n, thus exacerbating the winner's curse effect.

In independent private-values auctions, the winner's curse effect is absent, so *ceteris paribus* we should expect the markups to decrease as the number of participants increases, due solely to the competitive effect. In auctions where bidders' costs have both common-value and private-value components (as is the case in the model we employ), it is unclear which effect(s) will dominate: this is an empirical question, which can only be answered once one has estimates of the structural parameters in bidders' preferences.

Next, we describe the particular procurement setting which we study in this paper, and discuss issues related to applying the model of equilibrium bidding described in this section to this empirical setting.

3 New Jersey Department of Transportation construction services procurement auctions

In order to examine empirically the questions we have raised, we collected a data set of bids submitted in procurement contract auctions conducted by the New Jersey department of transportation (NJDOT) in the years 1989-1997. Over this period, the NJDOT conducted 1018 low-price, sealed-bid auctions of contracts to procure various services. For our empirical work, we focus on auctions for three types of jobs: highway work (worktype A in what follows), bridge construction and maintenance (worktype B), and road paving (worktype C), which together account for over half of the contracts auctioned during the sample period. See table 1 for summary statistics for these auctions.

[12] This is only true for common-value models, but not for private-value models. In ongoing work (Haile, Hong, and Shum (2000)), we are using this insight as the basis of a formal test for the presence of common-value components, based upon recent developments by Guerre, Perrigne, and Vuong (2000) and Hendricks, Pinkse, and Porter (1999) in the nonparametric estimation of structural auction models.

Clearly, the variation in the average winning bid across types of contracts indicate that the jobs defined in these contracts are markedly different. For that reason, we estimate separate parameters for each type of contract in our empirical work.

[Table 1 about here.]

Summary statistics Table 2 presents statistics on the observed bids, broken down according to the number of actual bidders who participated in the auction. Most auctions have between 3-7 bidders, and the NJDOT did not impose an explicit reserve price.[13] Importantly, there is a generally increasing (but by no means monotonically so) trend between number of bidders and both submitted and winning bids. For example, the average bid in the worktype A auctions rises from about $1.8 million in 4-bidder auctions to over $10 million, in 10-bidder auctions. While this is consistent with the hypothesis that the winner's curse (which leads to more cautious — higher — bids) dominates the increased competition effect (which leads to more aggressive — lower — bids), a more likely and non-strategic explanation is that these auctions are characterized by selective participation and contract heterogeneity, so that larger projects (which, from a purely cost perspective, command larger bids) attract more bidders. This suggests that controlling for contract heterogeneity is crucial to measuring the effects of the winner's curse, and we discuss this issue further below.

[Table 2 about here.]

Before proceeding to discuss specification issues, two remarks about the equilibrium bidding model we have just described are in order.

3.1 Contractor reimbursement schemes: is competitive bidding appropriate?

The contracts offered by the transportation department are characterized by contractor reimbursement guidelines which have both *cost-plus* (government assumption of all cost overruns above the bid) and *fixed price* (contractor assumption of all cost overruns above the bid) features. While each contractor submits a scalar bid b, he is also required as part of the bid submission process to break down this amount into a vector of unit prices $\vec{p}$ and an associated vector of tasks $\vec{q}$ which he agrees to perform for the payment $b = \vec{p}' \cdot \vec{q}$. While bids are ranked (and contracts awarded) solely on the basis of the scalar bid b, the contract details (unit prices and quantities) are important at the contract fulfillment stage.[14] The government assumes all cost increases due to unexpected increases in job quantities $\vec{q}$, but these cost increases are priced out at the unit prices bid by the contractors so that the contractor is bound to supply its services at its submitted unit prices $\vec{p}$; losses arising from unanticipated cost increases are *not* borne by the government.

These compensation guidelines raise two concerns. First, a moral hazard problem arises since contractors may not keep their costs as low as possible; second, contractors have little incentice to submit

[13] We return to the issue of an *implicit* reserve price later, when we consider the possibility that the actual number of bidders observed in these auctions may not correspond to the *potential* number of bidders.

[14] Helpful conversations with Marty Miller at the NJDOT clarified these institutional details.

cost-based bids for contracts where their eventual payoff is independent of their costs. McAfee and McMillan (1987) point out that the first problem will not affect the bidding process, since any rents from post-contractual opportunism should be competed away in equilibrium.

Institutional features of the procurement process tend to alleviate the second problem of noncompetitive bidding. Repeated interactions render reputational effects important in this procurement setting.[15] While these reputational effects are not a part of our formal model, we feel that the potential loss of future bidding eligibility may counteract contractors' incentives to submit bids which are non-indicative of their costs. In addition, contractors' costs are monitored on a fairly regular basis (every few weeks, from conversations with NJDOT officials). The original submitted bid must already indicate clearly projected materials costs, labor costs, and labor hours required. Deviations from these estimates must be rigorously justified.[16]

Most convincingly, however, the raw data seems to support the competitive bidding hypothesis. For a subset of the auctions we study, we were also able to obtain data on the *ex-post* compensation. The regression equation of the natural logarithm of compensation[17] on the natural logarithm of the winning bids is:

$$\begin{aligned} \log(\text{compensation}) = \begin{pmatrix} 0.0080 \\ (0.226) \end{pmatrix} + \begin{pmatrix} 1.0158 \\ (0.0166) \end{pmatrix} \log(\text{winning bid}) \\ R^2 = 0.928,\ N = 291 \end{aligned} \tag{3}$$

indicating no systematic overruns (insignificant constant) and a strong correlation between compensation and the bid (as is consistent with competitive bidding). In fact, cost *underruns* — which are inconsistent with any post-contractual opportunism scenario — occurred in 132/291 of these auctions. In any case, no evidence of systematic underbidding, which would be reflected in systematic overruns, exists.[18]

3.2 Is symmetry assumption appropriate?

A strong assumption of our model is symmetry: that the marginal joint distribution of (C_i, X_i) is identical across bidders i. In this section we point out reasons why symmetry may be violated, and present some statistical evidence appearing to confirm the symmetry assumption in our dataset.

[15] Many of the contractors in these auctions bid on many contracts over time, and likely derive a large part of their revenues from doing contract work for the state. In our data, we observe 421 distinct bidders, and each firm submits bids in an average of around 15.86 auctions in our data set. Successful bidders are even more active: in our sample, firms which are awarded at least one contract bid in an average of 29.43 auctions.

The NJDOT maintains a list of "prequalified contractors" which all firms must be on in order to be eligible to bid; given that the government observes *ex-post* compensation from all contracts, it is likely that firms who are judged to have acted opportunistically will be struck off the list. We did not have access to this list, which (naturally) changed over the course of the sample period, due mostly to entry of new contractors.

[16] The standard text by Halpin and Woodhead (1998) contains examples of typical cost-plus contracts in appendix E, as well as a description of cost control/monitoring techniques which are widely used in practice in chapter 14.

[17] Note that we never observe a firm's actual costs: the basis for firms' compensation are the per-unit costs submitted as part of the bid, which presumably already include a margin above actual costs.

[18] In the context of the bidding example above, however, under-runs occur chiefly because a job has required less time or materials than the contractor originally anticipated in submitting its bid.

How could asymmetries arise? Bajari (1998) has focused on asymmetries arising from differences in geographical location across contractors. Jofre-Bonet and Pesendorfer (1999) have examined a model in which capacity constraints introduce *intertemporal* linkages in contractors' bidding behavior which likewise induce asymmetries between contractors. Since we abstract away from both these sources of asymmetries in this paper, we attempt in this section to test our symmetry assumption.[19]

We exploit the panel nature of our data set (the availability of observations of bids by the same contractor submitted across different auctions) to test explicitly the symmetry hypothesis. More precisely, under our assumption of *ex ante* symmetry, each contractor i in an n-bidder auction identically wins the auction with probability $\frac{1}{n}$. Let $T_{n,i}$ denote the number of n-bidder auctions in which contractor i bids, in our data set; if these auctions are independent over time (which we also assume), the expected number of n-bidder auctions that contractor i wins is $\tilde{W}_{n,i} \equiv \frac{T_{n,i}}{n}$. If $W_{n,i}$ denotes the actual number of n-bidder auctions won by contractor i, then $X_{n,i} \equiv \left(W_{n,i} - \frac{T_{n,i}}{n}\right)$ is approximately distributed normal with zero mean and variance $\frac{T_{n,i}(n-1)}{n^2}$.

These tests were run for each bidder, and the empirical distribution of the p-values (i.e., upper tail probabilities under the null of symmetry) reported in table 3. Across all worktypes, the large p-values (for the top panel, the median p-values are 0.62, 0.65, and 0.62 for worktypes A, B, and C, respectively) suggest that we would not want to reject the null of symmetry. Furthermore, the same results hold even when we restrict the sample to observations where $T_{n,i} \geq 5$ (since the normal approximation of a binomial random variable is generally valid only for a large number of trials).[20]

[Table 3 about here.]

In this section, then, we have tested and cannot reject the hypothesis that the *ex ante* probability of winning any auction is identical across all participants. Our test has most power against alternatives which imply that this probability differs across contractors: these include the capacity constraints and geographical asymmetries emphasized in the papers cited above.[21]

4 Specification and estimation

Next, we describe the specific parameterization of contractors' costs which we employ.

[19] Both Bajari (1998) and Jofre-Bonet and Pesendorfer (1999) assume an independent private-values information structure, which simplifies the analysis of bidder asymmetries greatly.

[20] Although not reported here, the same results obtain when we further restrict $T_{n,i} \geq 10$. In this case, however, there are not many observations left.

[21] Despite the convincing nature of these statistical results, we raise the important caveat that collusion — which we abstract away from in this paper — can also lead, in equilibrium, to firms' winning equal proportion of the auctions in which they participate. In a symmetric bid rotation collusion scheme (similar to that considered by Porter and Zona (1993)) involving a full cartel where the non-winning bidders submit "phantom bids" which have no probability of winning, each contractor wins $\frac{1}{n}$ of the auctions. If there are T auctions, then, each bidder wins a fraction $\frac{T}{n}$ of them, which is observationally equivalent to an environment where each bidder faces an identical probability of winning. However, the large number of bidders vying for each type of contract during the sample period (cf. column 5 of table 1) would appear to render full cartels very difficult.

4.1 Specification details: the Wilson log-additive model

We follow Wilson (1998) in choosing a log-additive form for the cost function C_i.[22] We employ a symmetric version of Wilson's log-additive model, where contractor i's cost C_i is assumed to take the form

$$C_i = A_i \times W,$$

where A_i is a bidder i's private cost from undertaking the project (which could includes differences in labor efficiency between firms), and W is an unknown cost component which is common across all bidders (including, for example, uncertainty in future materials costs). In other words, C_i is the product of a common value (W) part and a private value (A_i) part.[23]

We assume that W and the A_i's are independently log-normally distributed: letting $\tilde{W} \equiv \log W$ denote the natural logarithm of W, and $\tilde{A}_i \equiv \log A_i$, then

$$\tilde{W} = m + \epsilon_w \sim N(m,\ \sigma_w^2)$$
$$\tilde{A}_i = \bar{a} + \epsilon_{a_i} \sim N(\bar{a},\ \sigma_a^2).$$

Each bidder is assumed to have a noisy signal of her cost of fulfilling the contract terms, X_i, which has the form

$$X_i = C_i \times E_i.$$

Here X_i is a contractor i's noisy estimate of the unknown cost C_i, and $E_i = \exp\{\sigma_e \xi_i\}$ where ξ_i is an (unobserved) error term that has a normal distribution with mean 0 and variance 1. If we let $\tilde{C}_i \equiv \log C_i$ and $\tilde{X}_i \equiv \log X_i$ then, conditional on $\tilde{C}_i$, $\tilde{X}_i = \tilde{C}_i + \epsilon_{e_i} \sim N(\tilde{c}_i,\ \sigma_e^2)$. Since m and $\bar{a}$ always appear together as a sum in this manner, we will not be able to estimate both parameters, but just their sum $\mu \equiv m + \bar{a}$.[24]

The relative magnitudes of the σ's indicate the relative importance of common-value and private-values components in bidders' preferences. As σ_e tends to 0, bidders' uncertainty about their costs disappears, and the model resembles a pure (affiliated) private values model. As σ_a tends to 0, the importance of the idiosyncratic component in bidders' valuations falls, implying a pure common value model. As σ_w tends to 0, bidders' uncertainty about the common component w disappears, making

[22] See also Wilson (1983), in which a similar specification was considered for a symmetric first-price auction.

[23] Given that C_i represents firm i's costs, there is a natural interpretation of the common component W as an index of unknown future input prices, and A_i as a "quantity index" of inputs (where the amount of each input required depends on firms' efficiency levels). Standard assumptions (cf. (Varian, 1992, ch. 9)) on the production technology enable one to aggregate the inner product of vectors of inputs and input prices as the product of a single price and quantity index.

[24] m, the mean of the prior distribution of w, could potentially include a "cost-padding" component which represents the bidders' common opportunities to engage in cost-inflation activities while undertaking the project. In this way, we accommodate moral hazard issues which are otherwise absent from our analysis. We feel that this is adequate since, as McAfee and McMillan (1987) note, equal cost-padding (or "shirking") opportunities across bidders will simply shift up bidders' costs by an equal amount, and not affect equilibrium bidding.

the model an independent private values model, but one in which bidders' imperfectly observe their private value (since $\sigma_e > 0$).[25]

A similar "log-additive" approach was also taken in the general "conditionally independent private information" models considered in Li, Perrigne, and Vuong (2000). Generally, both our approach and theirs employ the assumption that bidders observe only a *one-dimensional* signal X, although correlation among the signals across bidders is induced by a two-factor log-additive model. This assumption that the signal is one-dimensional is a crucial one which ensures tractibility of the model.[26]

4.2 Identification

Next we discuss identification of the model parameters, especially the crucial parameters $(\sigma_v, \sigma_a, \sigma_e)$ which indicate the important of common- and private-value components in bidders' preferences. Our large auction data set contains variation along two dimensions: bidders and number of participants. Our symmetry assumption implies that bidders are homogeneous, so that we do not exploit the fact that we observe an identical bidder across many bidding situations.[27] Variation in the number of participants across auctions drives heterogeneity in bidding behavior across contracts of a given worktype: for a given bidder, equilibrium bidding strategies in the low-price, sealed-bid auction model described in the previous section will differ depending on the number of rival bidders. Therefore, the distributions of the equilibrium bids will not be identical across auctions with different number of participants, even among all contracts of the same worktype. This is an important source of variation which we use to identify the parameters.[28]

However, this variation in N may not be wholly exogenous: for example, if contractors selectively participate in auctions of larger contracts, a spurious positive correlation between N and observed bids arises which could be misattributed to the effects of the winner's curse. Since we do not observe any measures of contract heterogeneity in our dataset, these considerations suggest that controlling for unobserved heterogeneity is crucial to measuring the effects of the winner's curse. We return to this problem later in this section.

[25] In short, this is a private value model where bidders don't observe their private value model; it can be turned into a standard PV model where we redefine the private value as $\mathcal{E}(c_i|x_i)$.

[26] In short, we assume that bidder i observes only one signal x_i, and cannot distinguish between its two components a_i and $w \times e_i$. A growing literature on auctions with multidimensional signals has shown that, outside of specific examples, it is difficult to characterize equilibrium in these situations. For example, Pesendorfer and Swinkels (2000) employ a model where a bidder's type consists of two quantities: a private value component and a (discrete) signal of the object's value. Indeed, they are only able to prove existence of ϵ-equilibria in this setting. The model of Wilson (1998) also assumes bidders to observe both a private value as well as a signal of their unknown valuation, but the equilibrium he describes hinges upon his assumption that bidders' have diffuse priors on the common value v, as well as log-normality. Finally, the difficulties of multi-dimensional types extends far beyond auctions into mechanism design in general; see Armstrong (1996) and Rochet and Stole (2000) for discussions.

[27] Such variation will be important once we investigate the asymmetric case, which is the topic of ongoing research.

[28] Given the sizeable variation in the number of bidders observed in the auction dataset, even for contracts of a given worktype, the possibility arises that bidders may not be aware of the number of rivals when they submit their bid. Hendricks, Pinkse, and Porter (1999) derive the equilibrium bidding strategies for a symmetric affiliated value auction. Below (in the appendix) we consider a specification which allows for this possibility.

It is generally difficult in very nonlinear parametric models to be very explicit about what types of variation in the data serve to identify particular parameters. Therefore we tackle the identification issue by simulating our model in order to investigate how changes in the parameter values affect the moments and quantiles of the $s_n(x;\theta)$ function, which form the basis of our estimation strategy. This exercise will alert us to parameters which are badly identified as those which spark no independent change in the (simulated) $s_n(x;\theta)$ function.

Graphs of $s_n(x;\theta)$ for alternate values of the parameters θ are given in figure 1. The benchmark values for the parameters $(\mu, \sigma_w, \sigma_a, \sigma_e)$ were $(-1, 1, 1, 1)$, and the graphs of $s_n(x;\theta)$ for these values, for 2,4,6,8 and 10 participants, is shown in the upper left-hand panel of figure 1. Note that the graphs are upward-sloping, as expected. More significantly, note that, for any given x, $s_n(x;\theta)$ is higher for larger number of participants: this demonstrates that for these parameters, the winner's curse (which encourages more cautious bidding as n increases) dominates the competitive effect (which encourages more aggressive bidding).[29]

Next, we explore how changes in the parameter values, relative to the benchmark case, affect $s_n(x;\theta)$. First, the lower right hand graph shows what happens to $s_n(x;\theta)$ in the pure private value case, as σ_e is set to 0. In this case, the equilibrium bid functions are decreasing in n, for a given x: as expected, in private-value auctions when the winner's curse is absent, bidding becomes more aggressive as n increases. A similar pattern results if we let $\sigma_w = 0$, as shown in the top right-hand side graph which, as discussed above, corresponds to an IPV model with an appropriate redefinition of the private value $\tilde{x}_x$ as $E[c_i|x_i]$. Finally, in the pure common value case (lower left hand side graph), in which σ_a is taken to zero, the $s_n(x;\theta)$ graphs resemble the benchmark graphs, but differ in magnitude.

As mentioned above, the variation in N may not be wholly exogenous in the presence of selective participation, and in figure 2, we explore in more detail the effects of σ_v, σ_a and σ_e on the equilibrium bids holding N fixed (at six), in order to address the identification of these parameters without relying on variation in N. Equilibrium strategies are computed for different combinations of the σ's. As we would expect, the effects of changing σ_a and σ_e differ depending on whether σ_v is far or close to zero. When $\sigma_v = 1$ (as in the top graph), the effect of an increase in σ_e is generally to increase bids at low values of the signals (the dotted-cross line, which has the highest value of σ_e, is the highest bid function), but this effect disappears for larger values of the signals. This is completely in line with the effects of the winner's curse, which we would expect to be more relevant at lower values of the signal (when a bidder's chances of winning are, *a priori*, small).

From the bottom graph, however, we see that when common values are not important (we set $\sigma_v = 0$), these winner's curse effects are not present at all. Here, note that the dotted-cross line

[29] The parameter identification strategy described in this section is based on comparisons of the *marginal* distributions of the equilibrium bids $s\left(F^{-1}(\tau;\theta);\theta\right)$ for different parameter values θ. As discussed in Donald and Paarsch (1996), this approach is particularly useful in independent private value models. For the affiliated value model we consider, the *correlation* structure among bids also contains informations useful for identifying parameters, and this has not been exploited in our monotone quantile estimator. We leave this interesting avenue for future research.

remains low throughout, demonstrating that an increase in the signal's "noisiness" actually leads to more competitive bidding over the entire range of signals.

In short, these graphs have demonstrated that perturbations in the basic parameters in the model do lead to changes in the shapes and/or magnitudes of the equilibrium bid functions, which lead us to believe that, from a computational point of view, the parameters are indeed identified and estimable.

[Figure 1 about here.]

[Figure 2 about here.]

4.3 Estimation approach: monotone quantile estimator

Equation (2) shows how assumptions on the joint distribution $F(x_1, v_1, \ldots, x_n, v_n; \theta)$ induces a joint distribution for the equilibrium bids, which we observe. Given that the distribution of $s_n(x;\theta)$ is likely to be quite asymmetric, even if we assume any individual x to be symmetrically distributed, we estimate the parameters via quantile restrictions, which try to match the "shapes" of the distributions of the observed bids and the $s_n(x;\theta)$.[30] As is well-known, quantile estimators are more robust to outliers in the data than estimators based on matching the centered moments.

Two insights drive our estimation procedure. First, the quantiles of a distribution $F(x)$ are invariant to monotonic transformations of the random variable x. Second, for our symmetric first-price auction, the equilibrium bidding strategies $s_{M_i}(x;\theta)$ for auction i (which had M_i participants) are monotonic transformations of the unobserved signals $x \sim F(x;\theta)$, where $F(\cdots)$ denotes the marginal distribution for a single signal.[31] In particular, $q_{\tau_k}^{M_i}(\theta)$, the τ_k-th quantile of the equilibrium bid function for the i-th auction, is just $s_{M_i}(x_{\tau_k}, \theta)$, the equilibrium bid function evaluated at x_{τ_k}, the τ_k-th quantile of the marginal distribution $F_{M_i}(x;\theta)$.[32]

Our estimator $\hat{\theta}$ minimizes the quantile objective function:

$$Q(\theta) = \sum_{i=1}^{T} \sum_{j=1}^{M_i} \sum_{k=1}^{K} \rho_{\tau_k} \left[b_{ij} - q_k^{M_i}(x_{\tau_k};\theta) \right]$$

where the function $\rho_{\tau_k}(\cdot)$ is defined as

$$\rho_{\tau_k}(x) = (\tau_k - 1(x \le 0))\, x.$$

[30] Recall that $\log X$ is normally distributed.

[31] For our log-normal model, $x_{\tau_k} = \exp\left(\sigma \Phi^{-1}(\tau_k) + \mu\right)$, where $\Phi(\cdot)$ is the standard normal cumulative distribution function, and μ and σ are, respectively, the mean and variance of $\log X_{\tau_k}$.

[32] The same insight appears implicit in the War of Attrition example considered in Milgrom and Weber (1985). Powell (1984) used the same intuition to derive a distribution-free least-absolute deviations (LAD) estimator for censored linear regression models: in his case, the monotonic censoring operation preserved the quantiles between the distribution of the additive error term and the (censored) dependent variable. (Manski, 1994, section 4.4) labels these "quantile independent monotone models".

There are two important features of this estimator. First, it alleviates (dramatically) the computational burden associated with simulating the moments of the equilibrium bid distribution.[33] Second, this estimator has a formal appeal since the quantile restrictions follow wholly from the theoretical result that the bidding strategies are monotonic transformations of the signals in equilibrium. Such an estimation approach is potentially very useful in other incomplete information settings where the equilibrium strategies (or the "policy functions") are monotonic transformations of the unobserved types: for example, nonlinear pricing (and, more generally, mechanism design) models appear very well-suited to this approach.[34]

5 Estimation results

Table 4 presents estimates of the four parameters of the benchmark model. In the discussion below, standard errors are enclosed in square brackets ($[\cdots]$).

Recall that the model resembles a private value model as σ_e tends to 0, and resembles a pure common value model as σ_a tends to 0. The small estimated σ_a for all three types of contracts (0.0024 [0.0038] for worktype A; 0.0064 [0.0084] for worktype B; and 0.0376 [0.1006] for worktype C) suggests that all these auctions can be described as predominantly pure common value auctions.[35]

[Table 4 about here.]

While striking, this finding is not wholly surprising, given the upward trend in both average and median bids observed in the raw data (cf. table 2). Abstracting away from all else, this upward trend can be attributed to the less aggressive bidding caused by the winner's curse, which only occurs in common value auctions. However, a more likely and non-strategic explanation is that these auctions are characterized by selective participation and contract heterogeneity, so that larger projects (which, from a purely cost perspective, command larger bids) attract more bidders. This suggests that controlling for contract heterogeneity is crucial to measuring the effects of the winner's curse, and next we explore specifications which explicitly accommodate this possibility by allowing the (parameters which characterize the) distribution of signals in auction i to depend explicitly on

[33] More precisely, to compute $s_{M_i}(x_{\tau_k};\theta)$, the τ_k-th quantile of the equilibrium bid distribution, the bid function need be evaluated only *once*, at x_{τ_k}. In contrast, in order to simulate $E_x s_{M_i}(x;\theta)$, the mean equilibrium bid, the bid function must be computed at S random draws from the marginal signal distribution, where S is the number of draws used to compute the simulated mean. Therefore, the monotone quantile estimator reduces the computational burden by an order of S by obviating the need to simulate the moments of the equilibrium bid distribution.

[34] In these models, the policy function $p(x)$ is often constrained to be monotonic in the type x in order to be implementable (i.e., satisfy incentive compatibility). See (Fudenberg and Tirole, 1991. 257ff.). More recently, Florens, Protopescu, and Richard (1997) have also developed a general estimation methodology for games of incomplete information, which requires inverting the equilibrium mapping of types to actions for each given set of parameter values. Our approach avoids this by exploiting the monotonicity of the equilibrium strategies and the invariance of distribution quantiles to monotonic transformations.

[35] In general, given the parameter restrictions that σ_a, σ_w, σ_e are all ≥ 0, it is not always straightforward to test whether any of the σ's is equal to zero, the lower bound of the parameter space. However, nonstandard tests need be employed only when testing joint hypotheses that two or more of the σ's are equal to zero (cf. Andrews (1998), Wolak (1989)); a standard one-sided t-test is valid for univariate tests.

M_i, the number of participants in auction i.[36]

[Table 5 about here.]

5.1 Robustness check: controlling for selective participation

First, we allow μ_i, the mean of the log-costs c_i in auction i, to have a quadratic trend in M_i, the number of participants in auction i:

$$\mu_i = \mu_0 + \mu_1 * M_i + \mu_2 * M_i^2 \tag{4}$$

and estimate μ_0, μ_1, and μ_2 as parameters. The results from these specifications are given in table 5.

Allowing the means to differ depending on the number of bidders does lead to changes in the parameter estimates for the important σ parameters. While we continue to reject the importance of private values in the worktype A auctions (σ_a is estimated to be 0.1294 [0.0824]), we can no longer do so for the auctions of the other types of contracts. The point estimate for σ_a in the auctions of worktype B and C contracts are much larger than before (1.1382 [0.1174] for worktype B, and 0.9778 [0.0078] for worktype C). From these results, we conclude that auctions for highway-work contracts are very close to a pure common-value auction, while both common-value and private-values are important in auctions of bridge repair and paving contracts.

The positive estimates of μ_1 for all three worktypes are consistent with the hypothesis that auctions with more bidders feature larger contracts; correspondingly, the negative estimate for the quadratic coefficient μ_2 for these worktypes indicates that this positive relationship only holds at smaller values of M_i.

5.2 Robustness check: Random effects for unobserved contract heterogeneity

The preceding specification is still restrictive because all contracts of the same worktype and with the same number of bidders are still assumed to be homogeneous. Unfortunately, we were able to collect contract characteristics for only a subset of the worktype A auctions,[37] and so in this section we focus on a specification which used random effects to control for unobserved heterogeneity across auctions.[38] More precisely, we assume that μ_i, the median of the signal distribution for auction i, is drawn from a normal distribution with mean η_i and standard deviation σ_{η_i}, and independent across auctions.[39] Furthermore, we parameterize η_i and σ_{η_i} as a function of the number of bidders M_i in

[36] While an explicitly structural model of contractors' participation decisions is beyond the scope of this paper, an model of bidder participation based on reserved price is discussed in the appendix.

[37] Results using these characteristics are presented in the appendix.

[38] This is a very common remedy in estimating duration models also, where unobserved heterogeneity can lead to spurious duration dependence (cf. Heckman and Singer (1984)).

[39] As suggested by a referee, we also decomposed the correlation among bids into the intra-auction and inter-auction components, in order to distinguish between the correlation among bids arising from common components that are observed by the bidders but not by the econometrician (unobseved heterogeneity), and the correlation that arises from common components unobserved by the bidders (a common value information structure). If the common component

auction i:

$$\eta_i = \gamma_0 + \gamma_1 * M_i + \gamma_2 * M_i^2$$
$$\sigma_{\eta_i} = \exp\left(\gamma_3 + \gamma_4 * M_i + \gamma_5 * M_i^2\right)$$

where $(\gamma_0, \ldots, \gamma_5)$ are parameters to be estimated. Note that this specification allows μ_i (the median signal for auction i) to be a *random* (across auctions, and from the econometrician's point of view) function of the number of bidders M_i. In the previous specification (equation (4)), the median bid μ_i is a deterministic function of M_i (and the same across all auctions with the same M_i).

[Table 6 about here.]

Table 6 contains parameter estimates from this specification. Instead of reporting estimates of $\gamma_1, \ldots, \gamma_5$ directly, we report the more easily interpretable estimates of η_M and σ_{η_M}, for $M = 1, \ldots, 15$.

Importantly, the above result that worktype A auctions can be described as almost pure common value auctions disappears in this set of results. For these auctions, it now appears that the variation in the private component (as parameterized by σ_a: 1.1938 [0.0236]) and the noisiness in the signals (measured by σ_e: 2.3048 [0.0444]) contribute more to the variation in the stochastic costs across bidders than uncertainty regarding the common component v (as parameterized by σ_w: 0.6741 [0.0148]). For worktype B, we continue to find that both private and common components are important, while for the worktype C auctions, we now find that σ_w, while marginally statistically different from zero (0.0634 [0.0335]), is very small in magnitude, making them similar to IPV auctions. Generally speaking, the worktype A contracts are longer in duration and larger in magnitude than the worktype B or C contracts, which support the finding that both CV as well as PV components are important in contractors' cost functions. Below, we will graphically consider the qualitative differences in these magnitudes in terms of predicted bids.

As for the unobserved heterogeneity parameters, the estimates indicate that for the worktype A contracts, the means of the heterogeneity distribution grow larger as the number of bidders increases, supporting the selective participation hypothesis (i.e., that larger contracts attract more contractors). However, the increasing values of σ_{η_M} suggest that there is a greater degree of heterogeneity in auctions attracting a larger number of bidders. While the results are qualitatively similar for worktype B, they are generally reversed for worktype C. Encouragingly, these results correspond to the patterns observed in the raw data (as given in columns 4 and 5 of table 2), suggesting that we are, indeed, adequately controlling for contract heterogeneity.

In the appendix, we consider two additional robustness checks: controlling for potential vs. actual

w is not observed by the bidders (as in an affiliated signals information structure), then bids are uncorrelated across auctions but correlated within auctions. If w is observed by the bidders but not by the econometrician, then the opposite could be true. For all three worktypes, we found that our measure of intra-auction correlation was much larger than the measure of inter-auction correlation, indicating that most of the correlation in the bids is within the auctions. This suggests that affiliated between the signals may be more important than unobserved heterogeneity in these auctions.

competition, and controlling for observed heterogeneity (for the worktype A auctions only). Since the results deom these specifications do not differ greatly from the random effects results from table 6, we will not discuss them here.

5.3 Model Fit

In section 4.2 above, we note that exogenous variation in the number of bidders aids in identifying the parameters of the model, given observations of the bids. By allowing the bid distribution to differ across the number of bidders, however, we rely on the parametric restrictions of our log-normal model to identify the parameters rather than exogenous variation in the data. Therefore, before proceeding, we check that our chosen parametric specifcation indeed fits the observed data.

In figure 3, we plot the actual and predicted median bids, by worktype and across different number of bidders. The predicted median bids were calculated from the estimation results reported in tables 5 and 6.[40] In general, the predicted and actual bids correspond reasonably closely, with the table 6 results fitting noticeably better.

[Figure 3 about here.]

The "hump" from 8–10 bidders in the actual worktype A auctions are missed by both specifications, but the predicted values are close at a smaller number of bidders. For the worktype B contracts, the table 6 fit remarkably close for auctions with less than 10 bidders. For the worktype C contracts, the table 6 fit reasonably well, across the whole range of cost signals.

In general, the fit of the model to the actual bids is reasonably close, and so next we perform bid simulations to address the effects of increasing competition on equilibrium bidding. In what follows, we focus on the table 6 results.

6 Increasing competition and equilibrium bidding

6.1 Increasing competition and individual bids

First, we examine how individual contractor bids would be potentially affected by increases in the number of competitors, i.e., whether $s_n(x)$ is increasing or decreasing in n, fixing the signal x. We are unaware of any general comparative static results. Previously, Smiley (1979) (chap. 3), Matthews (1984), and Pinkse and Tan (2000) have shown that the sign can go either way, depending on the parametric assumptions.[41] Since our information structure is more complicated than that considered

[40] For the table 6 results, we evaluate the bid functions at the mean random effect, i.e., assuming $\mu_i = \gamma_i$.

[41] Smiley (1979) gave examples of multiplicative bid functions where $s_n(x)$ decreases in n (i.e., more aggressive bidding), whereas Matthews (1984) focuses on the uniform distribution where $s_n(x)$ increases in n (i.e., less aggressive bidding). Pinkse and Tan (2000) characterize general conditions under which the latter occurs in affiliated *private-value* settings, with the striking implication that common values are not required to obtain equilibrium bidding behavior which becomes less aggressive as competition increases.

by these authors, we treat this as an open question which we address empirically.[42]

[Figure 4 about here.]

[Figure 5 about here.]

Equilibrium bid functions We simulate the equilibrium bidding function $s_n(x;\theta)$ function via formula (2) for the results from table 6. The graphs are shown in figure 4. In order to isolate the pure effect of the winner's curse, we zero out the coefficients attached to the number of bidders (i.e., $\gamma_1, \gamma_2, \gamma_4, \gamma_5$) in these simulations. Clearly, the shapes of the graphs vary noticeably over the three worktypes.

For worktypes A and B, the importance of uncertainty regarding a common component in costs is reflected in the fact that the equilibrium bid functions for these cases are increasing in the number of bidders n, across the range of signals; for instance, the equilibrium bid function evaluated at $\log x = -1.000$ rises from about $0.25 million with two bidders to over $1 million in ten-bidder auctions, a 400% percent increase; the magnitude of increase is even greater for the worktype B auctions.

[Figure 6 about here.]

This effect can arise due not only to the winner's curse, but also to PT's "affiliation effect". To gauge the importance of the latter effect, we re-simulate the bid functions setting $\sigma_e = 0$ (thus turning all the models into affiliated private-values models). The graphs, shown in figure 6, indicate that, for all three worktypes, once we make this change the bid functions are all decreasing in n, indicating that the affiliation effect is absent. As a result, we mostly attribute the increase in equilibrium bids shown in figure 4 to the winner's curse.

On the other hand, the graph for worktype C reflects the finding that privates values are important in these auctions. At low values of x, the equilibrium bids are falling in n, as we would expect in pure private value auctions. At large levels of x, however, some of the bid functions cross, indicating that the winner's curse seems to be more important in this range of x. For example, at $\log(x)$=-2.0, the worktype B equilibrium bid function for $n = 10$ lies below that for $n = 6$ (indicating that the competitive effect dominates), but at $\log(x) = -1.0$ the bid function for $n = 10$ has crossed over the $n = 6$ bid function. The winner's curse effect is stronger at larger value of x: for a given n, the probability of winning is smaller as signals get larger, which implies that as n increases the competitive effect will be relatively weaker upon bidders with larger signals, and winning an auction with a less optimistic signal (i.e., higher x) conveys worse news than winning with a more optimistic signal (i.e., lower x).

[42] Both Smiley and Matthews consider only the pure common value (or "mineral rights") model, whereas our model has both common-value and private-values components. Note that both authors also focus on high-bid auctions, where more aggressive bidding implies that $s_n(x)$ increases in n, and vice versa. The interpretations are reversed for our low-bid procurement auctions.

Equilibrium markups These results are clearly illustrated in terms of "markups" of the equilibrium bids in excess of expected costs which we may have expected naïve participants to bid for these contracts. Using our parameter estimates, we calculate the markup $\frac{b(x_i) - E[c_i|x_i]}{b(x_i)}$, where $E[c_i|x_i]$, the expected project cost to bidder i based just upon his signal x_i, is taken to be a "naive" estimate which bidder i might have bid were he incognizant about the winner's curse. The markups are plotted in figure 5.

For the worktypes A and B auctions, where uncertainty regarding the common component is important, the simulated markup is increasing in the n, across the entire range of x's. The markups themselves are quite large in magnitude: at $\log(x) = -1.0$ for the worktype A auctions, they increase from 10% with 2 bidders to around 80% with 10 bidders. This increase is reversed in part in the worktype C auctions, where we see that for smaller values of the signals, the markups are decreasing in n (indicating the strength of the competitive effect), while this trend is reversed at higher values of the signals, reflecting the informational effects due to the winner's curse.

[Table 7 about here.]

The difference in the magnitudes of the markups between the worktypes A/B and worktype C auctions is in large part attributable to winner's curse considerations. For the worktype C auctions, the common value element is negligible, so that the markups (at least in the lower range of the signals, when winner's curse considerations are not so important) are symptomatic of "market power" which recedes as the number of bidders increases. In the worktype A/B auctions, which have an important common value element, the markups arise to counter the adverse selection associated with winning the auction.

This point is illustrated in table 7, where equilibrium markups for the median bidder were calculated at different values of σ_e, which parameterizes the noisiness of contractors' signals regarding their unknown costs, with a larger σ_e corresponding to greater uncertainty. The markups were calculated after reducing σ_e by one-half (column 4) and then by 90% (column 5). Note that the reduction in uncertainty reduces the equilibrium markups for the worktype A auctions dramatically (for 6-bidder auctions, the markup falls from over 44% to 29.3% when σ_e is cut in half, and down to just 11.7% when σ_e is at one-tenth of its estimated value). This illustrates how a reduction in uncertainty reduces the winner's curse, and therefore the "premia" which bidders demand in this market. Similar trends are present for the worktype B auctions.

For worktype C auctions, however, a reduction in markups is not an obvious consequence of a reduction in uncertainty: for most n, the markups remain largely unchanged when going from $0.5 * \sigma_e$ to $0.1 * \sigma_e$. This emphasizes the point that when private value components in costs are important, winner's curse considerations are not so important in auctions with few bidders, so that equilibrium markups in these cases are attributable in large part to "market power". When the number of bidders increases, however, winner's curse effects become more important, and a reduction in uncertainty from σ_e to $0.5\sigma_e$ brings about a larger fall in equilibrium markups just as in the worktype A/B cases. Next, we explicitly explore what these effects imply about government procurement costs.

6.2 Increasing competition and project procurement costs

In procurement, common wisdom dictates that increasing the number of contractors would lower project costs. This is true for private value models but, given the results above (for worktypes A and B at least), one questions whether this is a wise policy for reducing project costs in common value auctions. Next, we present results from simulations of the winning bid in auctions in which the number of competitors is varied. We do this for the results in table 6, and graph the simulated bids in figure 7.

[Figure 7 about here.]

For the worktype A results, the average of the simulated winning bids is generally increasing in n, indicating that procurement costs would rise if the government invites more competition. The median winning bid rises from about $0.20 million with two bidders to $0.35 million once 10 bidders are involved, and continues to increase for $n > 10$. Furthermore, these simulations indicate that the "optimal" number of participants (which would minimize expected procurement costs) would be 3, which half of 6, the mode of the empirical distribution of n for the worktype A auctions (cf. table 2). Our results suggest that the government could lower expected procurement costs by about 15% by reducing n from 6 to 3 which, considering the average contract outlay of about $5 million, constitute substantial savings.

Opposite results are obtained for the worktype B and C results. For the worktype C results, which indicate a negligible common value component, the average winning bids fall quickly in n, from an average bid of $0.23 million with 2 bidders to about $0.18 million, with 10 bidders. This drop also appears in the worktype B results, but is much less precipitous. For these two types of contarcts, then, increasing competition would indeed lower procurement costs, and it is optimal for the government to invite as many tenders as possible.

Some well-known results (cf. Wilson (1977), Milgrom (1979)) exist concerning the limiting behavior of the winning bid in first-price auctions, as the number of bidders grows large. [43] The simulated winning bids for the worktype A auctions indicate that, for the range of n which we observe in the data, the average winning bid is increasing at smaller values of n. This does not contradict the theoretical results on information aggregation since that literature has remained silent about whether the (non-stochastic) sequence of average winning bids converges *monotonically*, at the given

[43] Matthews (1984) provides an intuitive illustration of these limit arguments, for the specific case where the distribution of the signal x conditional on the common value w is uniform in the interval $[0, w]$. More recently, Pesendorfer and Swinkels (1997) have shown that these conditions specified by Wilson (1977) and Milgrom (1979) can be weakened if one allows for multiple-unit auctions. In related work (Hong and Shum (2000), Chernozhukov, Hong, and Shum (2000)), we derive general conditions for information aggregation in common value auctions and, more generally, Bayesian "quantile experiments" in which the conditioning events are order statistics.

parameter values.[44]

7 Conclusions

We empirically measure the effects of the winner's curse on equilibrium bidding in procurement auctions. At common-value auctions, the winner's curse is an adverse selection problem which, in equilibrium, counsels more conservative bidding as the number of potential competitors increases. First, we estimate the structural parameters of an equilibrium bidding models to investigate the importance of common-value components in bidders' preferences. Second, we use these estimates to simulate hypothetical equilibrium bidding strategies as we increase the number of auction participants. We measure the effects on increasing competition on both individual bids as well as on its implication on procurement costs.

We analyze bid data from construction procurement auctions run by the New Jersey department of transportation in the years 1989-1997. Our results show that different types of contracts differ significantly in the degree that private and/or common value components are important, and these have contrasting implications on the effects of increasing competition on equilibrium bids and expected government procurement outlays. Both common value and private value are important in auctions of highway-work contracts and bridge repair contracts, while auctions of general paving contracts are closer to the independent private-values paradigm. Furthermore, our results indicate that the winner's curse is particularly strong in these highway and bridge repair contract auctions. Furthermore, winning bid simulations indicate that the average procurement cost is strictly increasing in the number of bidders as competition intensifies: for example, the median costs rise about 15%, as the number of bidders is increases from 3 to 6. These results emphasize how asymmetric information can overturn the common economic wisdom that more competition is always desirable, and have potentially important policy implications.

Methodologically, we have estimated a model which allows bidders' latent valuations for a contract to have both common-value and private-values components. To our knowledge, this is the first empirical implementation of a model with such flexible bidder preferences.

The obvious policy implication here is that governments may wish to restrict entry, or favor "negotiations" over auctions (cf. Bulow and Klemperer (1996)) when the winner's curse is particularly strong. But in practice, this may not be always feasible, since government procurement agencies try to reduce the possibility of collusion among contractors by inviting more tenders (i.e., increasing competition).[45] Our findings, while not directly addressing these issues, do contain a striking impli-

[44] In results which we have not reported, we simulated sequences of winning bids for all three worktypes, and found that while convergence obtains for B and C type contracts, the sequence of winning bids for worktype A contracts appears to diverge.

[45] For example, a recent tender of auto-towing contracts in Toronto was scrapped due to low levels of participation while municipal staff were "instructed [...] to come back with suggestions on how more companies might be able to participate" (*Toronto Star* (1999)). In general, collusion and bid-rigging seem rampant in procurement settings, for example, Pesendorfer (2000); Baldwin, Marshall, and Richard (1997); and Porter and Zona (1993) for studies of

cation: in situations where the winner's curse is so severe as to lead to *higher* procurement costs as the number of bidders increases (as in the worktype A results in table 7), municipal authorites may actually prefer to *allow* collusion, since in a common value setting the informational pooling that arises from bidder discussions may defuse the winner's curse effects. This is one potential justification for why the US government allowed joint bidding in the Outer Continential Shelf offshore lease auctions from their inception in the 1950s until the mid-1970s (cf. Hendricks and Porter (1992), Hendricks and Porter (1996)). The potential benefits of such restrictions on competition have been noted previously in the theoretical literature by, among others, Bulow and Klemperer (1999), and we leave a more thorough analysis to future research.

several instances.

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A Details on estimation

A.1 Simulation details

For the Wilson log-normal specification, $\mathcal{E}\left(C_i|x_1,\ldots,x_n\right)$ can be expressed in analytic form. Before deriving this, we introduce some notation. In this model, the vector $(\tilde{c}_1 = \log c_1,\ldots,\tilde{c}_n = \log c_n, \tilde{x}_1 = \log x_1,\ldots,\tilde{x}_n = \log x_n)$ is distributed as jointly normal with identical means μ for all elements and variance-covariance matrix $\hat{\Sigma} = ((\Sigma,\Sigma_{12}),(\Sigma_{12}',\Sigma^*))'$.[46] Furthermore, given the above log-normality assumptions, the conditional expectation functions for c_i take the following form:

$$E[C_i \mid x_1,\ldots,x_n] = \exp\left[E(\tilde{c}_i \mid \tilde{x}_1,\ldots,\tilde{x}_n) + \frac{1}{2}Var(\tilde{c}_i \mid \tilde{x}_1,\ldots,\tilde{x}_n)\right], \tag{5}$$

for $i = 1,\ldots,n$.

Next, we denote the marginal variance-covariance matrix of $(\tilde{c}_1,\tilde{x}_1,\ldots,\tilde{x}_n)$ by

$$\Sigma_i \equiv \begin{pmatrix} \sigma_c^2 & \sigma_c^{*\prime} \\ \sigma_c^* & \Sigma^* \end{pmatrix}$$

where $\sigma_c^2 = \sigma_w^2 + \sigma_a^2$ is the variance of c_i. Using the normality assumption

$$E(\tilde{c}_i \mid \tilde{x}) = \mu + \sigma_c^*\Sigma^{*-1}(\tilde{x}-\mu) \tag{6}$$

and

$$Var(\tilde{c}_i \mid \tilde{x}) = \sigma_c^2 - \sigma_c^{*\prime}\Sigma^{*-1}\sigma_c^*. \tag{7}$$

Combining expressions (6) and (7) with equation (5), given parameter estimates, the conditional expectation (5) can be explicitly evaluated for every vector of log-signals $(\tilde{x}_1,\ldots,\tilde{x}_n)$.[47]

Given the analytic expressions for the conditional expectations (5), the next step is to calculate the conditional expectations (8). Recall that the vector of log-signals $(\tilde{x}_1,\ldots,\tilde{x}_n)$ is jointly-normal with mean vector $M^* = (\mu^*,\ldots,\mu^*)$ and variance covariance matrix Σ^*. Let $\tilde{x}_{3+}$ denote the sub-vector of log-signals $\tilde{x}_3,\ldots,\tilde{x}_n$, and $\tilde{x}_{+2}$ denote the vector of log-signals $\tilde{x}_1,\tilde{x}_2$. Then

$$\begin{pmatrix} \tilde{x}_{3+} \\ \tilde{x}_{+2} \end{pmatrix} \sim \text{normal}\left(\begin{bmatrix} \mu \\ \mu \end{bmatrix}, \begin{bmatrix} \Sigma_{3+}^* & \Sigma_{3+,+2}^{*\prime} \\ \Sigma_{3+,+2}^* & \Sigma_{+2} \end{bmatrix}\right)$$

[46] In estimation we use explicit formulas for the elements in the matrix $\hat{\Sigma}$ given the information structure of the model.

[47] Consequently, the desired conditional expectations can be obtained by simulation; viz.,

$$v(x,x) = \mathcal{E}\left(c_i|x_i = x, \min_{j\neq i} x_j = x\right) =$$

$$\underbrace{\int\cdots\int}_{x_k \geq x,\ \forall k=3,\ldots,n} \mathcal{E}\left(c_1|x_1,\ldots,x_n\right)\ dF\left(x_3,\ldots,x_n|x_1 = x, x_2 = x, x_k \geq x, k = 3,\ldots,n;\theta\right) \tag{8}$$

where F here denotes the conditional distribution of the signals $x_3,\ldots,x_n$, conditional on $x_1 = x_2 = x$. Given symmetry, there is no loss of generality in focusing on the pair of signals x_1 and x_2.

where the elements of the mean vector and variance-covariance matrix can be determined from the information structure of the model.

Then, again using the multivariate normal conditional expectation formulas:

$$\tilde{x}_{3+}|\tilde{x}_{+2} \sim \text{normal}\left(\mu + \Sigma^{*}_{3+,+2}{}'\Sigma^{-1}_{+2}(\tilde{x}_{+2} - \mu)\ , \Sigma^{*}_{3+} - \Sigma^{*}_{3+,+2}{}'\Sigma^{-1}_{+2}\Sigma^{*}_{3+,+2}\right) \tag{9}$$

Thus, given log-signals $(\tilde{x}_1, \tilde{x}_2)$, we can take S draws of $\tilde{x}_{3+}$ according to the conditional distribution (9) and simulate the integral in (8). In the results below, we utilized the GHK algorithm for drawing from truncated multivariate distributions. Essentially, this algorithm "importance samples" recursively from the truncated region, one dimension at a time, and allows for smooth (in θ) simulation of the required conditional expectation $v_n(x, x; \theta)$. While this simulated estimate of the conditional expectation is biased for a finite number of draws, some Monte Carlo evidence (cf. Keane (1994), McFadden and Ruud (1995)) has demonstrated that this bias is small in practice.[48] Let R_2 denote the number of simulation draws used in simulating $v_n(x, x; \theta)$.

A.2 Asymptotic distribution for quantile estimator

Pakes and Pollard (1989) derive a general asymptotic theory for estimators obtained by maximizing simulated objective functions.At this point, we utilize expressions for the variance-covariance matrices of the estimators which do not take account of the possible simulation bias due to using a finite number of draws. The implicit assumption, then, is that the number of simulation draws used in the various stages of computation increase faster than $\sqrt{T}$, the rate at which $\hat{\theta}$ converges to the true $\hat{\theta}_0$.

In what follow, we assume that θ is L-dimensional. Out quantile estimator minimizes the quantile objective function, reproduced here as

$$Q(\theta) = \sum_{i=1}^{T}\sum_{j=1}^{M_i}\sum_{k=1}^{K}\rho_{\tau_k}\left[b_{ij} - q_k^{M_i}(\theta)\right]$$

where M_i denotes the number of bidders in auction i, $q_k^{M_i}(\theta)$ denotes the kth quantile of bids for an M_i-bidder auction, and $\rho_{\tau_k}(\cdot)$ is defined as

$$\rho_{\tau_k}(x) = (\tau_k - 1(x \leq 0))\,x.$$

At $\hat{\theta}^{SQ}$, the SQ estimator, the approximate first-order condition must hold:

$$\frac{1}{\sqrt{T}}\sum_{i=1}^{T}\sum_{j=1}^{M_i}\sum_{k=1}^{K}\left\{\tau_k - 1\left[b_{ij} \leq q_k^{M_i}\left(\hat{\theta}^{SQ}\right)\right]\right\}\frac{\partial q_k^{M_i}\left(\hat{\theta}^{SQ}\right)}{\partial\theta} = o_p(1)$$

[48] We used a version of the GHK code for multivariate normal distribution available from V. Hajivassiliou's website: `www.lse.ac.uk/vassilis`.

It can be shown (cf. (Gourieroux and Monfort, 1995, chap. 8.5.2)) that

$$\sqrt{T}\left(\hat{\theta}^{SQ}-\theta\right)=-A_T^{-1}B_T+o_p(1)$$

where

$$B_T=\frac{1}{\sqrt{T}}\sum_{i=1}^{T}\sum_{j=1}^{M_i}\sum_{k=1}^{K}\left\{\tau_k-1\left[b_{ij}\leq q_k^{M_i}(\theta_0)\right]\right\}\frac{\partial q_k^{M_i}(\theta_0)}{\partial\theta}$$

and

$$A_T=\frac{1}{T}\sum_{i=1}^{T}\sum_{j=1}^{M_i}\sum_{k=1}^{K}f_{\tau_k}^{M_i}\left[q_k^{M_i}(\theta_0)\right]\frac{q_k^{M_i}(\theta_0)}{\partial\theta}\frac{q_k^{M_i}(\theta_0)'}{\partial\theta'}$$

where $f_{\tau_k}^{M_i}$ is the density of the marginal distribution of the random variable $s_{M_i}(x;\theta)$ in a M_i bidder auction at the τ_kth quantile, where x denotes the signal.

We estimate the A_T matrix using a finite difference approximation for the density $f_{\tau_k}^{M_i}$ as well as the gradient $\frac{q_k^{M_i}(\theta_0)}{\partial\theta}$. In this way, the (d_1,d_2) element of the $L\times L$ A_T matrix will be

$$\hat{A}_T^{rs}=\frac{1}{4Th_T^2}\sum_{i=1}^{T}\sum_{j=1}^{M_i}\sum_{k=1}^{K}\left\{1\left[b_{ij}\leq q_k^{M_i}\left(\hat{\theta}^{SQ}+h_Te_{d_1}\right)\right]-1\left[b_{ij}\leq q_k^{M_i}\left(\hat{\theta}^{SQ}-h_Te_{d_1}\right)\right]\right\}* \left[q_k^{M_i}\left(\hat{\theta}^{SQ}+h_Te_{d_2}\right)-q_k^{M_i}\left(\hat{\theta}^{SQ}-h_Te_{d_2}\right)\right] \tag{10}$$

where e_d is a L-vector with 1 in the dth position and zeros otherwise, and h_T is the perturbation factor for the finite-difference approximation.

Via a central-limit theorem (using independence over auctions i), the L-vector B_T has an asymptotic distribution $N(0,V)$. A consistent estimator of V is

$$\hat{V}_T=\frac{1}{T}\sum_{i=1}^{T}\hat{m}_i(b_i)\,\hat{m}_i(b_i)' \tag{11}$$

where

$$\hat{m}_i(b_i)=\sum_{k=1}^{K}\sum_{j=1}^{M_i}\left(\tau_k-1\left(b_{ij}\leq q_k^{M_i}\left(\hat{\theta}^{SQ}\right)\right)\right)\frac{\partial q_k^{M_i}\left(\hat{\theta}^{SQ}\right)}{\partial\theta}.$$

The gradient vector $\frac{\partial q_k^{M_i}(\hat{\theta}^{SQ})}{\partial\theta}$ can likewise be evaluated using finite difference methods.

Based on equations 10 and 11, the asymptotic variance-covariance matrix for $\hat{\theta}^{SQ}$ can be approximated by

$$\frac{1}{T}\hat{A}_T^{-1}\hat{V}_T\hat{A}_T^{-1}.$$

B Additional specifications

B.1 Robustness check: Potential competition vs. actual competition

Next we consider a model which allows for the possibility that N, the number of actual bidders observed in the data, does not equal $\mathcal{N}$, the number of potential bidders. We present a model where the difference between $\mathcal{N}$ and N arises from a reserve price r, which we assume to be constant across all auctions (and is a parameter to be estimated).[49]

We utilize a model of equilibrium bidding in first-price auctions with a reserve price, as in Milgrom-Weber, section 7. Define the screening value

$$x_n^*(r) = sup\left\{x|E\left[V_1|X_1 = x, Y_1 > x\right] \leq r\right\} \tag{12}$$

where we have subscripted by n, the number of bidders. Note that the right-hand side of this expression is a "participation constraint" which defines the highest participating signal.

Note that the entry cost does not affect the differential equation at all, only the boundary conditions. The boundary condition is now $b^*(x_n^*(r)) = r$, and the equilibrium bid function should be

$$b_n^*(x;r)\begin{cases} = L\left(x,x^*\right) * r + \int_x^{x^*} v(\alpha,\alpha)dL(x,\alpha) & \text{if } x < x_n^*(r) \\ > r & \text{otherwise}\end{cases}$$

where

$$L\left(x,y\right) = \exp\left(-\int_x^y \frac{f_{Y_1}(s|s)}{1-F_{Y_1}(s|s)}ds\right).$$

Note that $L(x,x) = 1$.

After doing the integration by parts, we get the simpler expression

$$b_n^*(x;r)\begin{cases} = \left(r - v\left(x_n^*(r), x_n^*(r)\right)\right) * L\left(x, x_n^*(r)\right) + v\left(x,x\right) + \int_x^{x_n^*(r)} L\left(x,\alpha\right) v'\left(\alpha,\alpha\right) d\alpha & \text{if } x < x_n^*(r) \\ > r & \text{otherwise}\end{cases} \tag{13}$$

Note that

$$r = E\left[V_1|X_1 = x_n^*(r), Y_1 > x_n^*(r)\right] \geq E\left[V_1|X_1 = x_n^*(r), Y_1 = x_n^*(r)\right] = v\left(x_n^*(r), x_n^*(r)\right)$$

so that the first term is positive.

[49] As noted above, there was no explicit reserve price in these auctions. However, in the model proposed in this section, we rely on an implicit reserve price to generate differences between the *potential* and *actual* number of bidders. Note that, without a reserve price, there is no reason in our model why all qualified bidders would not participate in each and every auction (we don't consider capacity constraints or other intertemporal linkages in this analysis). Furthermore, entry or bid preparation costs would not discourage entry in our *low-bid* auctions (as they do in the high-bid auctions considered in Hendricks, Pinkse, and Porter (1999) or Milgrom and Weber (1982), section 7; they would simply lead bidders to "pad" their bids by the amount of the entry costs.

In order to simulate the equilibrium bid function, we need an estimate of r. Assuming that r is constant across all auctions of a given worktype, a natural estimate of r is $\max_{i,j} b_{ij}$, the highest bid observed in the data. As is well-known (cf. Donald and Paarsch (1996)), this estimate is super-consistent, and its estimation does not affect the asymptotic distribution of our GMM estimator. Given this, in each iteration of the estimation process, we need to compute the screening values $x_n^*(r)$ for $n = 1, 2, \ldots$ via the implicit relation (12). Given these screening values, we can compute the equilibrium bid function (13).

Furthermore, given our parametric assumption and the parameter values, we can derive the probability that out of $\mathcal{N}$ potential bidders, N submit bids:

$$\begin{aligned} P_{N,\mathcal{N}} &\equiv \text{Binomial}\left(N, \mathcal{N}, \Phi\left(\log x_{\mathcal{N}}^*(r)\right)\right) \\ &= \binom{\mathcal{N}}{N} \left[\Phi\left(\log x_{\mathcal{N}}^*(r)\right)\right]^N \left[1 - \Phi\left(\log x_{\mathcal{N}}^*(r)\right)\right]^{\mathcal{N}-N} \end{aligned} \tag{14}$$

where $\Phi(\cdot)$ denotes the marginal CDF of the log-signals (a normal distribution with mean μ_n and some variance).

Since $\mathcal{N}$ is unobserved by the econometrician, we treat it as a random variable which, conditional on the observed number of bidders N, we assume to follow a multinomial probability law:

$$Prob\left(\mathcal{N}|N\right) = \frac{P_{N,\mathcal{N}}}{\sum_{\mathcal{N}'=N}^{N+5} P_{N,\mathcal{N}'}}, \; \mathcal{N} = N, N+1, \ldots, N+5. \tag{15}$$

For an auction with N observed bidders, we simulated bids by drawing an $\mathcal{N}$ according to (15), and then calculating the equilibrium bid correspond to this $\mathcal{N}$ using the top equation in (13) above. The results from these specifications are given in table 8. The magnitudes of the important σ parameters do not differ greatly from the random effects estimates from table 6, and for that reason we will not discuss them further.

[Table 8 about here.]

B.2 Robustness check: Accounting for contract-specific heterogeneity

In our last robustness check, we control for observed heterogeneity across contracts by parameterizing μ_i, the median of the signal distribution for auction i, as a function not only of M_i, the number of bidders in auction i, but also the covariates specific to auction i. The two most important dimensions of heterogeneity are size differences across contracts of a given worktype, and changes in input costs across time which also exogenously affect bids for a contract. Since our bid data do not allow us to determine the per-unit costs submitted by the bidders, adequately controlling for size heterogeneity is particularly important.

We were only able to obtain contract-specific covariates for the worktype A contracts. This is because many of these contracts specified roads upon which work was to be done. We obtained contract-specific covariates which were characteristics of these roads, as extracted from a database maintained by the NJDOT.[50] Table 9 below summarizes the covariates. They are: TRAFFIC, a measure of the weekday traffic volume (in both directions) of the road being repaired; and geographic dummies (GATEWAY, SKYLANDS, SHORE, DELAWARE, SOUTH) which describe the geographic location of the road. Furthermore, we also obtained construction cost indices corresponding to the month in which a particular contract was auctioned from the trade publication *Engineering News-Record* (NY is the index for New York City, and PHIL is the index for Philadelphia; in our specification, we use an average of the two).

[Table 9 about here.]

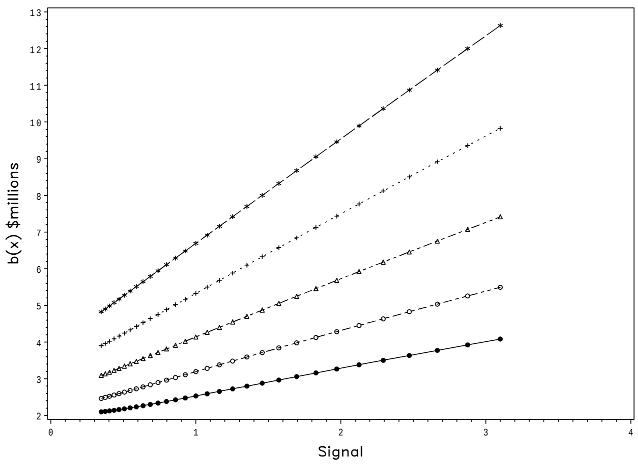
Using these covariates, we parameterize μ_i as:

$$\mu_i = \mu_0 + \mu_1 * M_i + \mu_2 * M_i^2 + \mu_3 * \left(\frac{1}{2} * \text{NY+PHIL}\right) +$$
$$\mu_4 * \text{TRAFFIC} + (\mu_5\ \mu_6\ \mu_7\ \mu_8\ \mu_9) \begin{pmatrix} \text{GATEWAY} \\ \text{SKYLANDS} \\ \text{SHORE} \\ \text{DELAWARE} \\ \text{SOUTH} \end{pmatrix}.$$

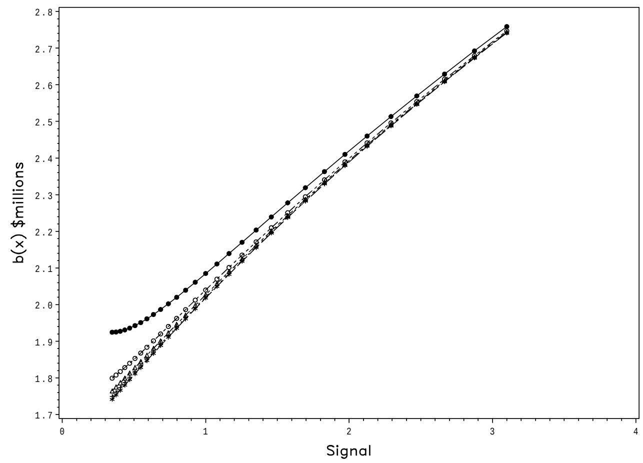
[Table 10 about here.]

Since we were only able to obtain covariates for the worktype A contracts, this specification was only estimated on those auctions. The results are reported in table 10. The magnitudes and signs of the parameters common across all the specifications remain quite stable (relative to the random effects specifications in table 6) to the incorporation of covariates. In particular, note that σ_e, which measures the noisiness of bidders' signals, continues to be large in magnitude (2.3400 [0.0072]), which implies that contractors' cost signals are very noisy, which tends to reinforce our finding of a strong winner's curse effect in these auctions (as we discuss below). Furthermore, most of the covariates enter significantly in the specification, and in the direction expected (for example, COST enters positively). Give the stability of the point estimates between the random effects and observed heterogeneity specifications, then, we will not discuss these results further.

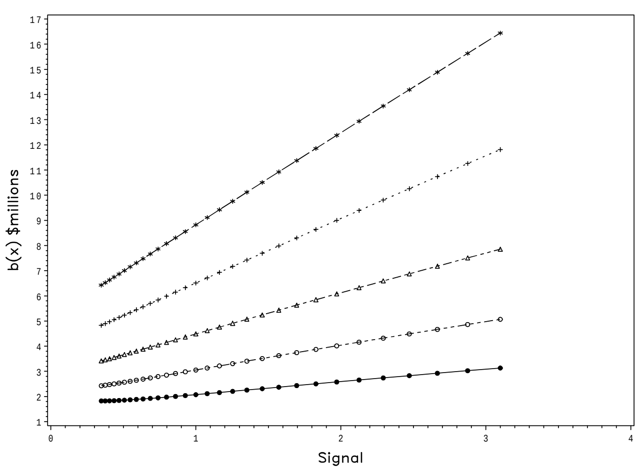
[50] This database can be searched on-line at `www.state.nj.us/transportation/count/search/search1.htm`.



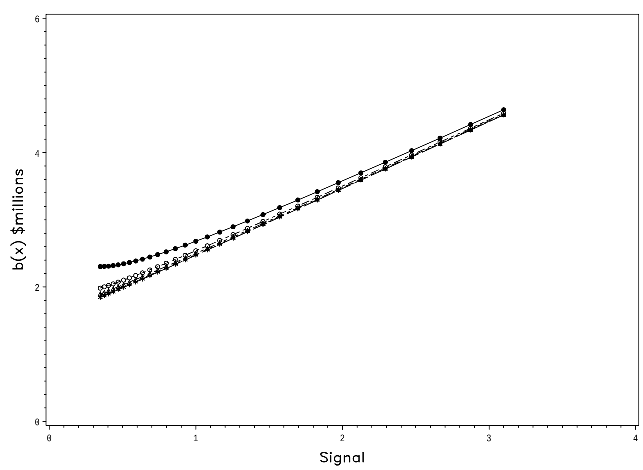
Baseline case: Parameters $(\mu, \sigma_v, \sigma_a, \sigma_e) = (1, 1, 1, 1)$



Parameters $(\mu, \sigma_v, \sigma_a, \sigma_e) = (1, 0, 1, 1)$



Parameters $(\mu, \sigma_v, \sigma_a, \sigma_e) = (1, 1, 0, 1)$: Pure common value



Parameters $(\mu, \sigma_v, \sigma_a, \sigma_e) = (1, 1, 1, 0)$: Pure private value

Figure 1: Plots of equilibrium bidding functions for different parameter values

Legend:

2 bidders: –●–●–

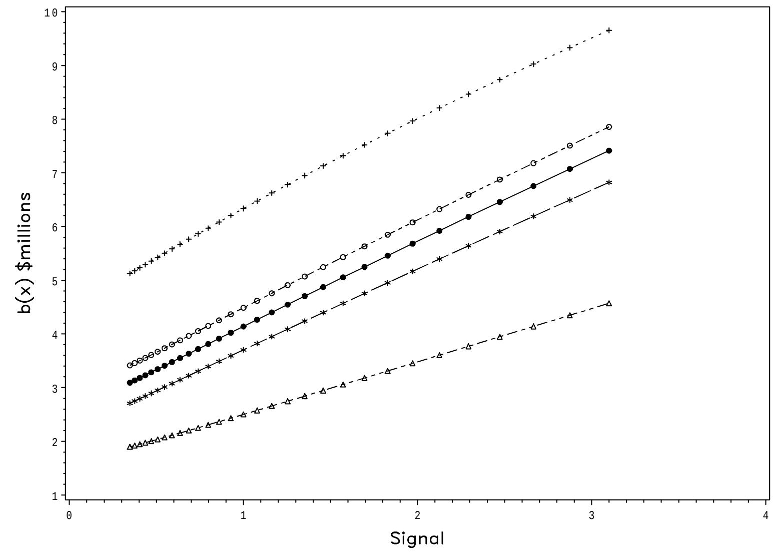
4 bidders: –○—○—

6 bidders: △- — –△–

8 bidders: · · + · · + ·

10 bidders: —-*—*—*

Figure 2: How bid functions vary in σ_a and σ_e, for $N = 6$



Legend:

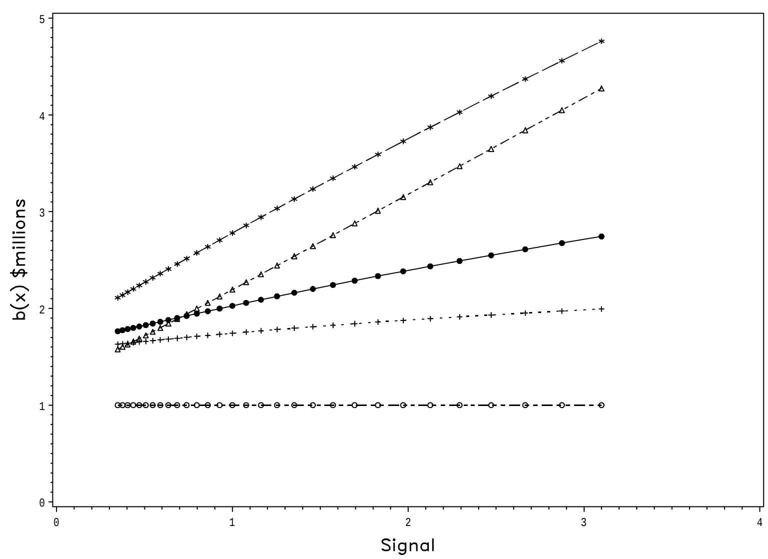
—●—●— $\sigma_v = 1,\ \sigma_a = 1,\ \sigma_e = 1$

—○—○— $\sigma_v = 1,\ \sigma_a = 0,\ \sigma_e = 1$

△- — –△– $\sigma_v = 1,\ \sigma_a = 1,\ \sigma_e = 0$

· · + · · + · $\sigma_v = 1,\ \sigma_a = 1,\ \sigma_e = 2$

—-*—*—* $\sigma_v = 1,\ \sigma_a = 2,\ \sigma_e = 1$



Legend:

—●—●— $\sigma_v = 0,\ \sigma_a = 1,\ \sigma_e = 1$

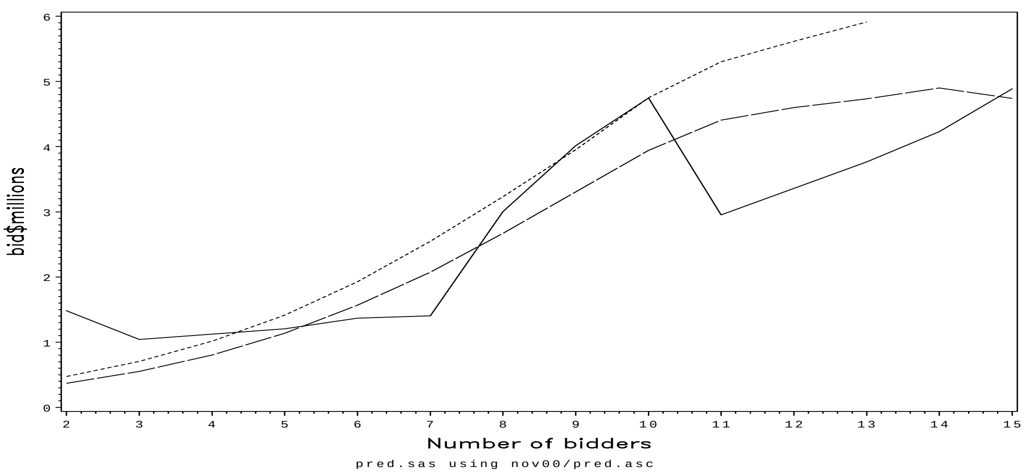
—○—○— $\sigma_v = 0,\ \sigma_a = 0,\ \sigma_e = 1$

△- — –△– $\sigma_v = 0,\ \sigma_a = 1,\ \sigma_e = 0$

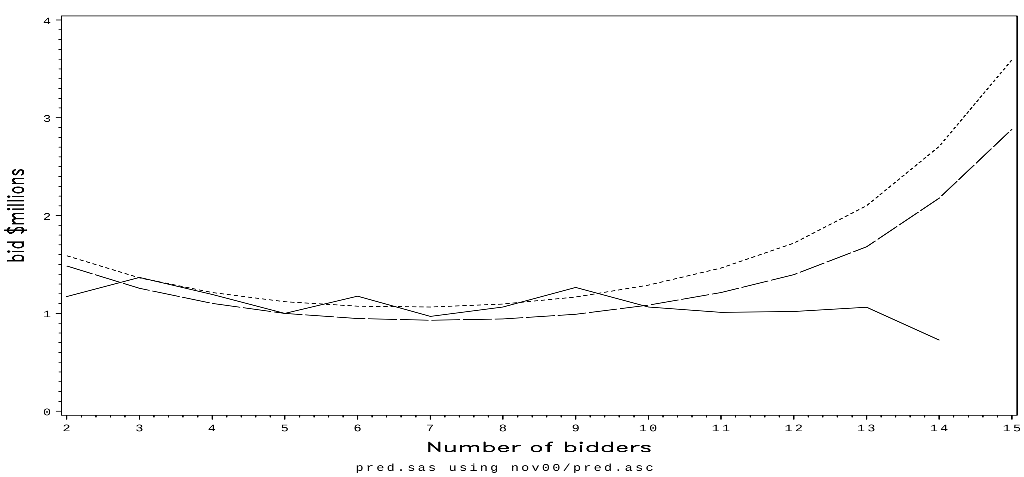
· · + · · + · $\sigma_v = 0,\ \sigma_a = 1,\ \sigma_e = 2$

—-*—*—* $\sigma_v = 0,\ \sigma_a = 2,\ \sigma_e = 1$

For worktype A contracts



For worktype B contracts



For worktype C contracts

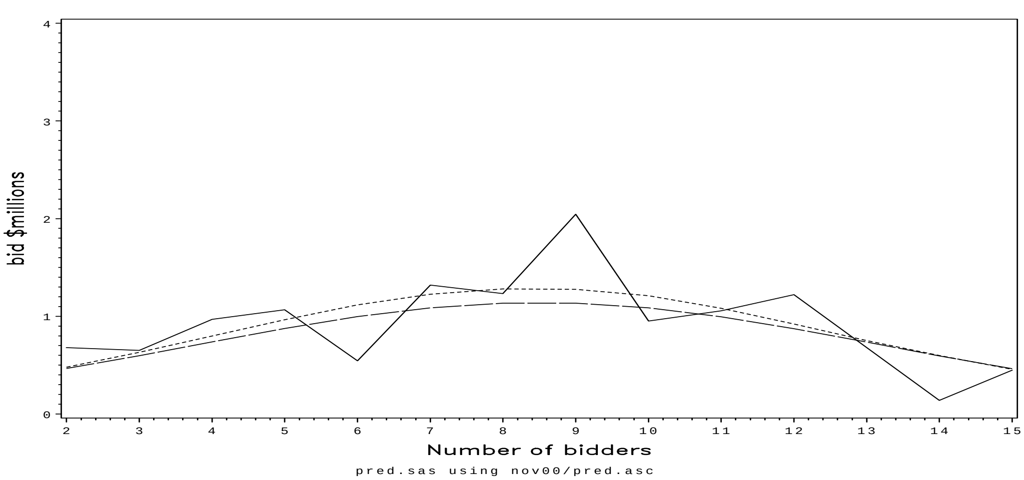
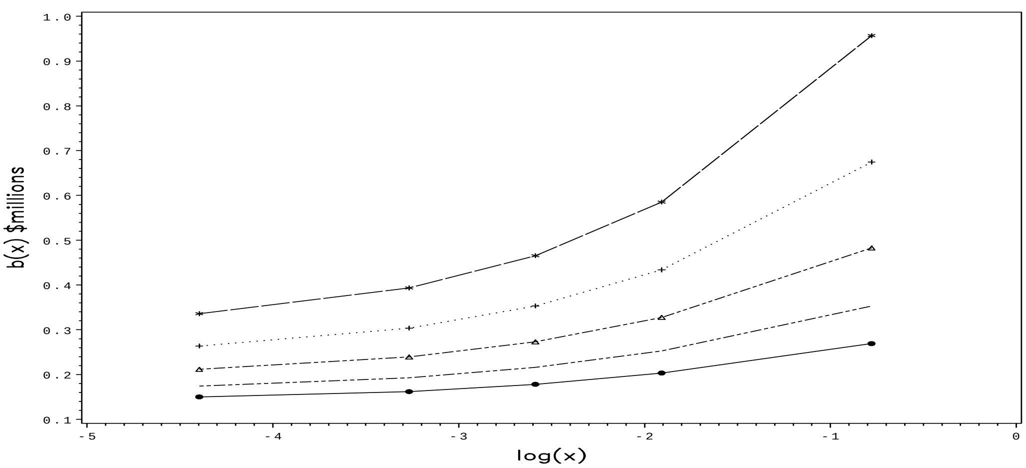


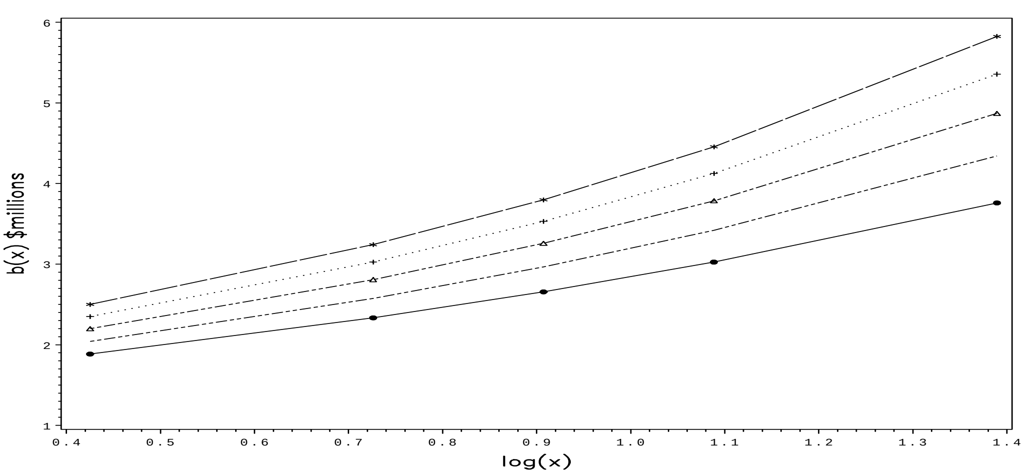
Figure 3: Predicted vs. actual bid functions: using table 5 and table 6 results

Legend: Actual median bid (from table 2; solid line)
Predicted median bid using table 5 results (dotted line)
Predicted median bid using table 6 results (dashed line)

For worktype A results



For worktype B results



For worktype C results

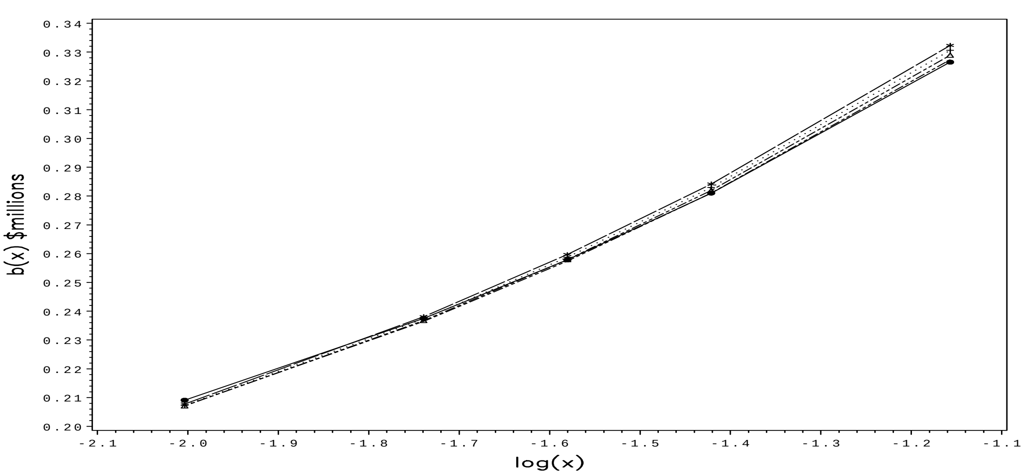
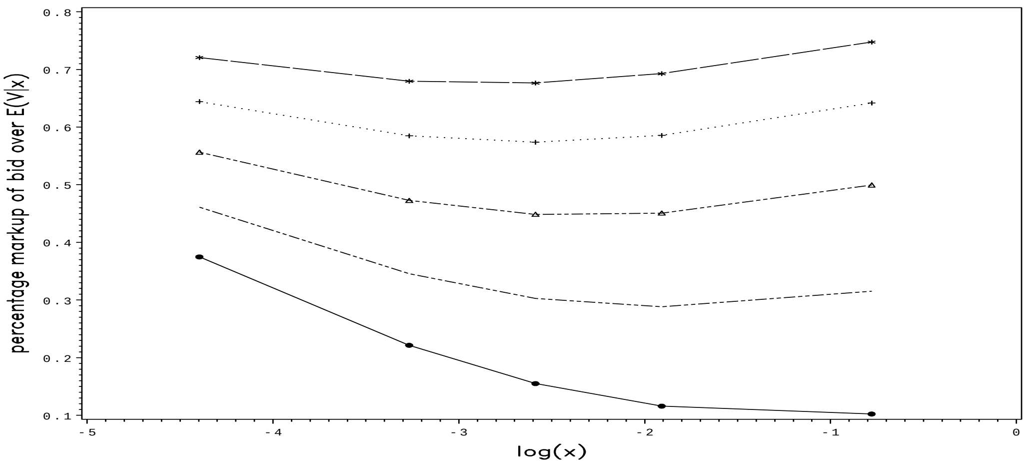


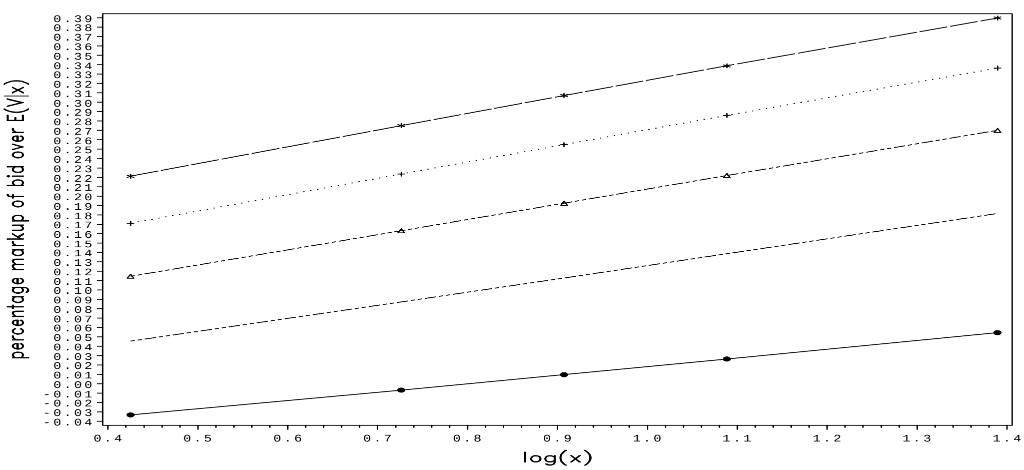
Figure 4: Equilibrium bid functions: simulated using table 6 results

Legend: –•–•– 2 bidders - - — - — 4 bidders △- — –△– 6 bidders · · + · · + · 8 bidders —-*—*—* 10 bidders

For worktype A results



For worktype B results



For worktype C results

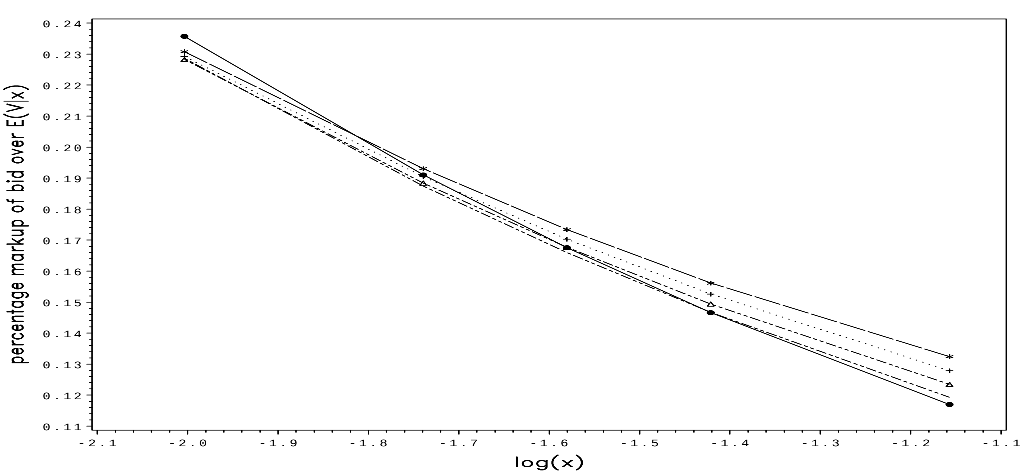
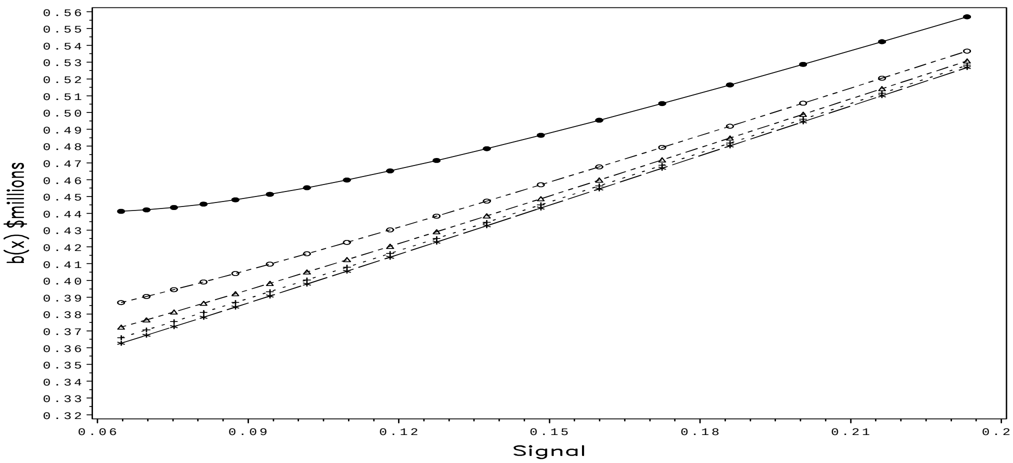


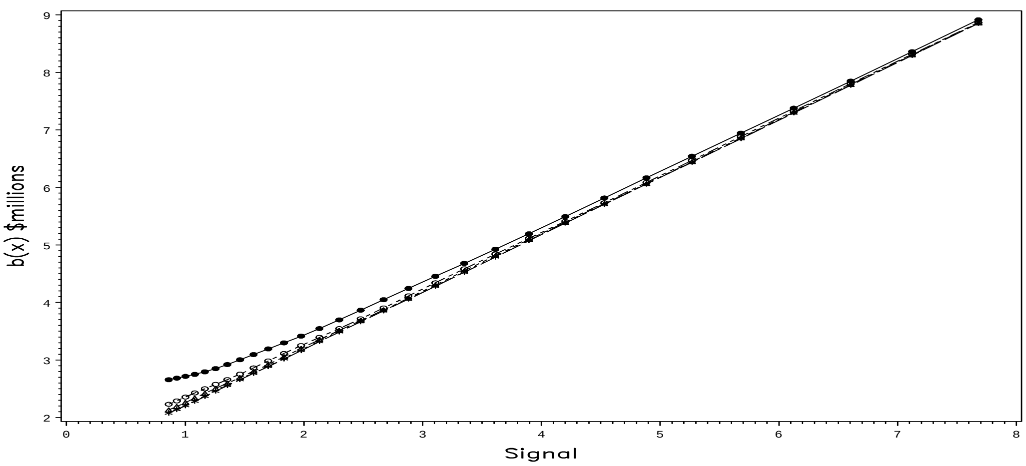
Figure 5: Bidder markups: simulated using table 6 results
Markups are given by $\frac{b-\mathcal{E}(c|x;\theta)}{b}$.

Legend: –•–•– 2 bidders – - — - — 4 bidders △- — –△– 6 bidders · · + · · + · 8 bidders —*—*—* 10 bidders

For worktype A results



For worktype B results



For worktype C results

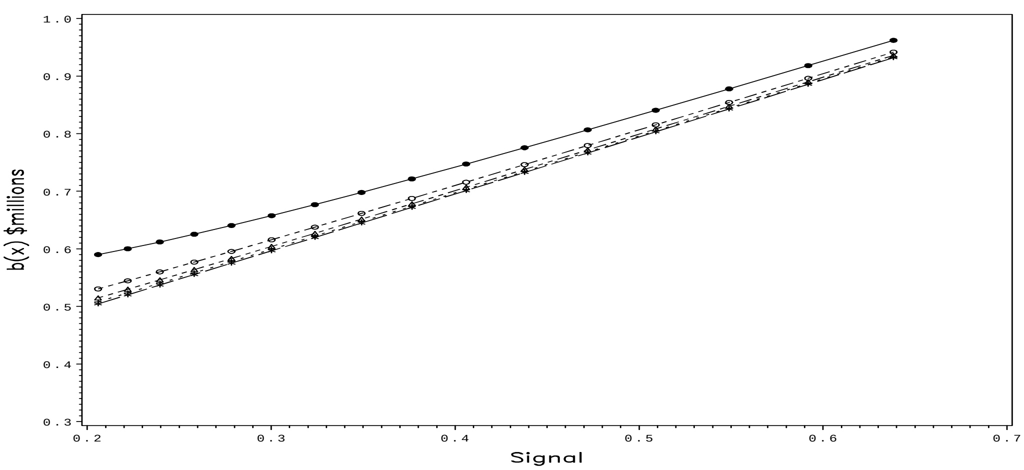
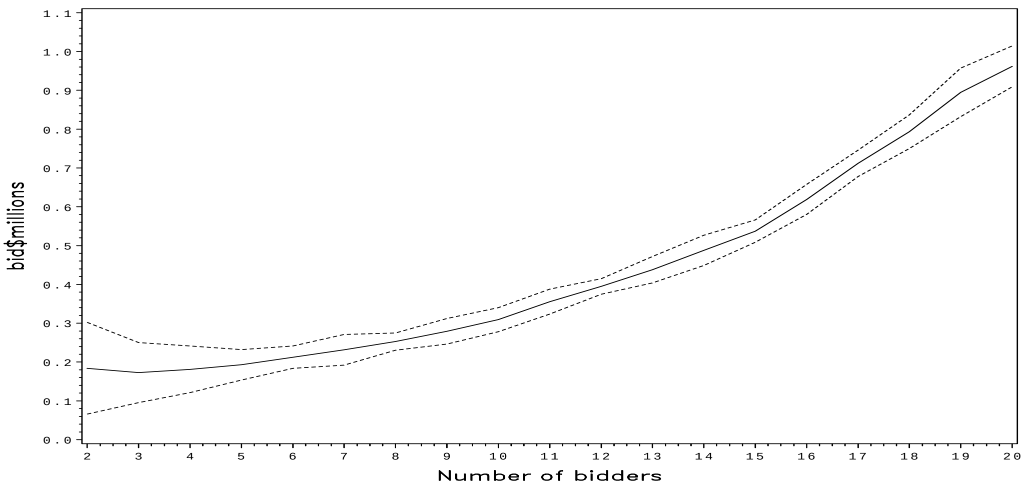


Figure 6: Equilibrium bid functions: Affiliation vs. Winner's Curse Effect?
Simulated using table 6 results.

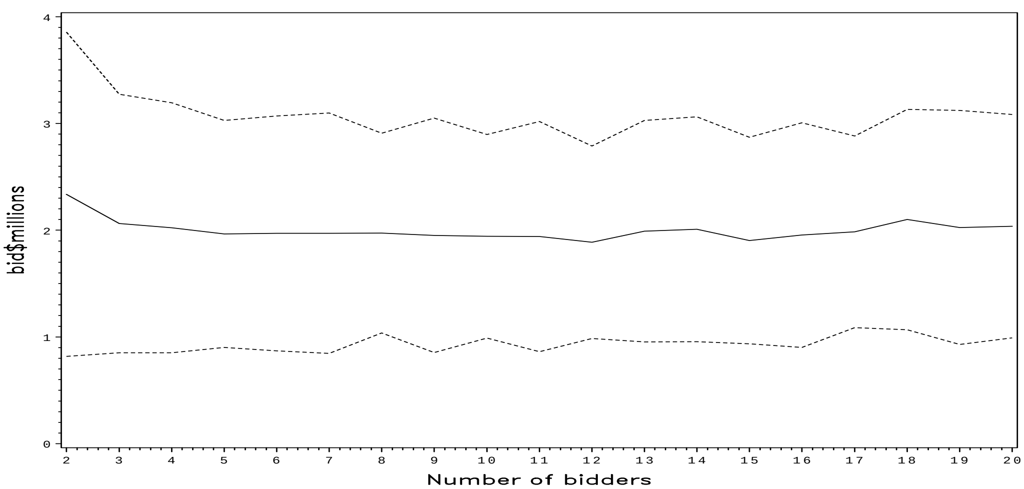
Legend: –•–•– 2 bidders – - — - — 4 bidders △- — –△– 6 bidders · · + · · + · 8 bidders —-*—*—* 10 bidders

Figure 7: Do procurement costs increase in the number of bidders?

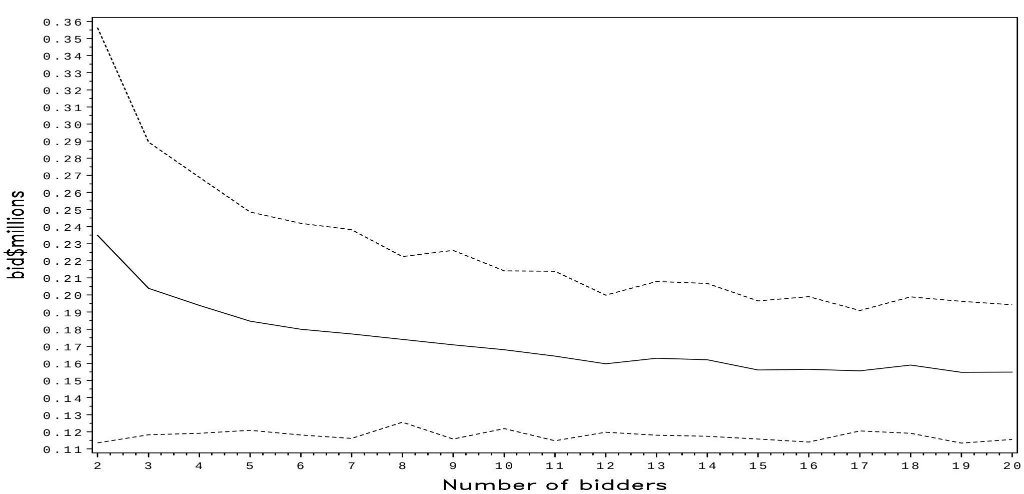
For worktype A results



For worktype B results



For worktype C results



Legend: Average winning bid in solid line
+2stdev and -2stdev in dashed lines

Table 1: Breakdown of auctions by job type

	Worktype	#auctions	Avg. winning bid (1989$, mil)	Stdev., winning bid	Total bidders	Total winners
A	General highway	423	4.97	9.61	274	121
B	Bridge construc./repair	194	1.48	1.82	171	74
C	Grading & Paving	150	0.97	1.11	109	33

Table 2: Summary statistics on bids

Worktype	Number of bidders	#auctions	Average bid (1989$; mils)	Std. dev.	Median bid (1989$; mils)	Average winning bid
A	2	12	5.894	14.954	1.482	5.601
A	3	31	1.692	2.000	1.042	1.520
A	4	46	1.843	1.919	1.124	1.605
A	5	51	3.380	5.223	1.204	3.015
A	6	58	4.513	8.310	1.369	3.982
A	7	46	4.435	9.751	1.406	3.526
A	8	40	6.365	12.567	3.000	5.229
A	9	39	8.658	15.438	4.016	6.640
A	10	22	10.612	12.828	4.745	9.256
A	11	20	15.087	30.432	2.951	11.471
A	12	17	11.704	10.935	7.739	10.263
A	13	12	10.652	14.879	3.767	8.984
A	14	8	10.523	12.264	4.229	9.350
A	15	8	9.274	10.338	4.885	8.274
A	16	3	2.506	1.053	2.477	1.544
A	17	4	9.999	5.029	9.830	8.583
B	2	12	1.265	0.651	1.171	1.167
B	3	7	1.577	1.244	1.368	1.432
B	4	24	1.672	0.495	1.193	1.386
B	5	12	1.049	0.730	0.999	0.819
B	6	23	1.566	1.265	1.175	1.286
B	7	23	1.278	1.120	0.968	1.001
B	8	16	2.644	3.496	1.064	2.241
B	9	19	1.755	1.439	1.266	1.349
B	10	14	1.480	1.245	1.065	1.141
B	11	7	1.149	0.811	1.010	0.888
B	12	10	1.932	1.899	1.019	1.398
B	13	13	2.261	2.457	1.063	1.869
B	14	5	1.784	1.496	0.728	1.462
B	15	3	8.472	6.032	6.953	6.873
B	16	5	2.658	2.816	1.254	1.952
B	17	6	3.056	3.825	1.352	2.415
C	2	9	0.737	0.418	0.679	0.664
C	3	21	0.629	0.308	0.650	0.546
C	4	23	1.086	0.624	0.968	0.910
C	5	34	1.566	2.266	1.068	1.347
C	6	27	0.826	0.674	0.543	0.705
C	7	19	1.469	1.011	1.320	1.273
C	8	8	0.961	0.608	0.963	0.789
C	9	4	2.424	0.939	2.045	2.059
C	10	5	1.087	0.518	0.951	0.955
C	11	4	1.164	0.801	1.056	0.844
C	12	1	1.231	0.098	1.221	1.105
C	14	1	0.152	0.057	0.138	0.094
C	15	1	0.453	0.049	0.448	0.374

Table 3: Test for symmetry

Empirical quantiles of p-value for H_0: $X_{n,i} \equiv \left(W_{n,i} - \frac{T_{n,i}}{n}\right) = 0$.

Worktype	#obs	Empirical quantiles of p-values 10%	25%	50%	75%	90%
Across all observations						
A	363	0.13361	0.41422	0.61708	0.73888	0.78151
B	793	0.08326	0.47950	0.65472	0.73888	0.78927
C	1190	0.08326	0.41422	0.61708	0.73888	0.78151
For observations where $\mathbf{T}_{n,i} \geq 5$						
A	42	0.036389	0.13361	0.31731	0.49624	0.88017
B	53	0.058782	0.31731	0.41422	0.51861	0.84148
C	178	0.062979	0.28009	0.42203	0.61209	0.86577

Table 4: Parameter estimates: baseline specification
Standard errors in parentheses

Worktype: Parameter:	A	B	C
σ_w	2.4529 (0.3883)	1.4877 (0.1280)	1.3811 (0.0523)
σ_a	0.0024 (0.0038)	0.0064 (0.0084)	0.0376 (0.1006)
σ_e	0.9771 (0.0007)	0.1612 (0.1075)	0.4317 (0.2191)
μ	-0.4261 (0.0309)	0.0694 (0.0190)	-0.4803 (0.2095)
Simulation draws:[a]	50	50	50

[a]Number of simulation draws used in calculating $v_n(x, x; \theta)$

Table 5: Parameter estimates
Control for selective participation: parameterize $\mu = \mu_0 + \mu_1 * n + \mu_2 * n^2$
Standard errors in parentheses

Worktype: Parameter:	A	B	C
σ_w	2.2621 (0.1898)	0.7847 (0.1038)	0.9054 (0.0349)
σ_a	0.1294 (0.0824)	1.1382 (0.1174)	0.9778 (0.0078)
σ_e	0.5196 (0.1675)	0.4030 (0.2445)	0.3622 (0.0740)
μ_0	-1.6814 (0.4086)	0.8266 (0.3000)	-1.6563 (0.3248)
μ_1	0.3891 (0.1143)	-0.2759 (0.0959)	0.4146 (0.0948)
μ_2	-0.0142 (0.0080)	0.0191 (0.0095)	-0.0258 (0.0063)
Simulation draws:[a]	50	50	50

[a]Number of simulation draws used in calculating $v_n(x, x; \theta)$

Table 6: Parameter estimates
Robustness Check: Random effects
Standard errors in parentheses

Worktype: Parameter:	A	B	C
σ_w	0.6741 (0.0148)	0.4583 (0.0900)	0.0634 (0.0335)
σ_a	1.1938 (0.0236)	0.3298 (0.1769)	0.5201 (0.0311)
σ_e	2.3048 (0.0444)	0.4383 (0.0818)	0.3455 (0.0618)
η_1	-2.5873 (0.0713)	0.9074 (0.1116)	-1.5803 (0.0312)
η_2	-1.7721 (0.0419)	0.3125 (0.0241)	-0.9035 (0.0174)
η_3	-1.4274 (0.0305)	0.0776 (0.0330)	-0.6331 (0.0130)
η_4	-1.1247 (0.0221)	-0.1155 (0.0669)	-0.4082 (0.0106)
η_5	-0.8639 (0.0177)	-0.2670 (0.0990)	-0.2287 (0.0102)
η_6	-0.6451 (0.0180)	-0.3768 (0.1275)	-0.0946 (0.0113)
η_7	-0.4682 (0.0213)	-0.4449 (0.1522)	-0.0059 (0.0130)
η_8	-0.3333 (0.0259)	-0.4713 (0.1728)	0.0375 (0.0153)
η_9	-0.2404 (0.0308)	-0.4560 (0.1896)	0.0354 (0.0181)
η_{10}	-0.1893 (0.0361)	-0.3990 (0.2024)	-0.0121 (0.0217)
η_{11}	-0.1803 (0.0418)	-0.3004 (0.2115)	-0.1049 (0.0264)
η_{12}	-0.2132 (0.0484)	-0.1600 (0.2169)	-0.2432 (0.0323)
η_{13}	-0.2880 (0.0562)	0.0221 (0.2189)	-0.4269 (0.0395)
η_{14}	-0.4048 (0.0656)	0.2459 (0.2176)	-0.6559 (0.0480)
η_{15}	-0.5635 (0.0766)	0.5113 (0.2136)	-0.9304 (0.0579)
σ_{η_1}	0.1553 (0.0062)	0.3661 (0.0101)	0.5943 (0.0096)
σ_{η_2}	0.1553 (0.0062)	0.3661 (0.0101)	0.5943 (0.0096)
σ_{η_3}	0.1691 (0.0064)	0.3675 (0.0099)	0.5817 (0.0081)
σ_{η_4}	0.1856 (0.0065)	0.3718 (0.0094)	0.5689 (0.0069)
σ_{η_5}	0.2052 (0.0067)	0.3790 (0.0087)	0.5560 (0.0060)
σ_{η_6}	0.2288 (0.0069)	0.3893 (0.0079)	0.5428 (0.0056)
σ_{η_7}	0.2571 (0.0072)	0.4028 (0.0070)	0.5296 (0.0057)
σ_{η_8}	0.2911 (0.0077)	0.4200 (0.0064)	0.5162 (0.0062)
σ_{η_9}	0.3323 (0.0086)	0.4413 (0.0067)	0.5027 (0.0069)
$\sigma_{\eta_{10}}$	0.3823 (0.0101)	0.4672 (0.0087)	0.4892 (0.0077)
$\sigma_{\eta_{11}}$	0.4434 (0.0126)	0.4983 (0.0124)	0.4757 (0.0086)
$\sigma_{\eta_{12}}$	0.5182 (0.0166)	0.5356 (0.0178)	0.4621 (0.0096)
$\sigma_{\eta_{13}}$	0.6104 (0.0226)	0.5800 (0.0251)	0.4486 (0.0105)
$\sigma_{\eta_{14}}$	0.7248 (0.0315)	0.6329 (0.0346)	0.4350 (0.0114)
$\sigma_{\eta_{15}}$	0.8674 (0.0442)	0.6958 (0.0467)	0.4216 (0.0122)
Simulation draws:[a]	50	50	50

[a]Number of simulation draws used in calculating $v_n(x, x; \theta)$

Table 7: Equilibrium markups: the effect of reduction in uncertainty

Worktype	# bidders	Markup: σ_e	Markup: 0.5*σ_e	Markup: $0.1 * \sigma_e$
A	2	0.155	0.168	0.256
A	3	0.227	0.181	0.198
A	4	0.303	0.209	0.159
A	5	0.374	0.246	0.133
A	6	0.449	0.293	0.117
A	7	0.511	0.336	0.108
A	8	0.574	0.382	0.103
A	9	0.628	0.425	0.101
A	10	0.676	0.467	0.103
A	11	0.712	0.495	0.105
A	12	0.736	0.513	0.108
A	13	0.764	0.537	0.115
A	14	0.799	0.582	0.128
A	15	0.823	0.612	0.143
B	2	0.010	0.036	0.041
B	3	0.066	0.070	0.053
B	4	0.113	0.099	0.064
B	5	0.153	0.123	0.074
B	6	0.192	0.144	0.082
B	7	0.226	0.161	0.087
B	8	0.255	0.174	0.090
B	9	0.281	0.183	0.089
B	10	0.307	0.190	0.085
B	11	0.320	0.189	0.079
B	12	0.326	0.184	0.070
B	13	0.335	0.179	0.060
B	14	0.364	0.182	0.051
B	15	0.380	0.181	0.041
C	2	0.168	0.118	0.129
C	3	0.166	0.091	0.100
C	4	0.166	0.075	0.081
C	5	0.167	0.064	0.068
C	6	0.168	0.057	0.060
C	7	0.169	0.053	0.055
C	8	0.170	0.051	0.053
C	9	0.172	0.052	0.053
C	10	0.173	0.055	0.056
C	11	0.175	0.060	0.061
C	12	0.176	0.068	0.069
C	13	0.178	0.080	0.082
C	14	0.179	0.098	0.101
C	15	0.180	0.124	0.128

Table 8: Parameter estimates
Robustness Check: Potential competition vs. actual competition
Standard errors in parentheses

Worktype: Parameter:	A	B	C
σ_w	0.9625 (0.0002)	0.0995 (0.0003)	0.0133 (0.0002)
σ_a	2.5029 (0.0072)	0.2115 (0.0010)	0.6319 (0.0013)
σ_e	1.3393 (0.0020)	0.6618 (0.0003)	0.1538 (0.0010)
η_1	-2.5979 (0.0098)	2.0491 (0.0078)	-1.9606 (0.0050)
η_2	-1.7019 (0.0098)	1.3089 (0.0073)	-1.2616 (0.0028)
η_3	-1.2705 (0.0102)	0.9913 (0.0072)	-0.9659 (0.0015)
η_4	-0.8501 (0.0109)	0.7086 (0.0072)	-0.7060 (0.0007)
η_5	-0.4407 (0.0118)	0.4608 (0.0073)	-0.4820 (0.0022)
η_6	-0.0424 (0.0129)	0.2480 (0.0074)	-0.2939 (0.0042)
η_7	0.3448 (0.0142)	0.0702 (0.0076)	-0.1416 (0.0064)
η_8	0.7210 (0.0155)	-0.0728 (0.0079)	-0.0251 (0.0089)
η_9	1.0862 (0.0170)	-0.1807 (0.0083)	0.0555 (0.0115)
η_{10}	1.4403 (0.0186)	-0.2537 (0.0088)	0.1002 (0.0144)
η_{11}	1.7834 (0.0202)	-0.2918 (0.0093)	0.1091 (0.0174)
η_{12}	2.1154 (0.0219)	-0.2949 (0.0099)	0.0821 (0.0207)
η_{13}	2.4363 (0.0237)	-0.2630 (0.0106)	0.0193 (0.0241)
η_{14}	2.7463 (0.0256)	-0.1963 (0.0114)	-0.0794 (0.0278)
η_{15}	3.0451 (0.0274)	-0.0945 (0.0123)	-0.2139 (0.0317)
σ_{η_1}	0.4314 (0.0019)	0.3403 (0.0004)	0.7867 (0.0007)
σ_{η_2}	0.4314 (0.0019)	0.3403 (0.0004)	0.7867 (0.0007)
σ_{η_3}	0.4648 (0.0021)	0.3468 (0.0004)	0.7689 (0.0006)
σ_{η_4}	0.5017 (0.0024)	0.3575 (0.0004)	0.7506 (0.0006)
σ_{η_5}	0.5424 (0.0027)	0.3727 (0.0004)	0.7317 (0.0006)
σ_{η_6}	0.5875 (0.0031)	0.3929 (0.0005)	0.7125 (0.0006)
σ_{η_7}	0.6374 (0.0035)	0.4190 (0.0005)	0.6929 (0.0006)
σ_{η_8}	0.6928 (0.0041)	0.4519 (0.0006)	0.6730 (0.0007)
σ_{η_9}	0.7543 (0.0047)	0.4929 (0.0007)	0.6529 (0.0007)
$\sigma_{\eta_{10}}$	0.8227 (0.0054)	0.5437 (0.0008)	0.6326 (0.0007)
$\sigma_{\eta_{11}}$	0.8988 (0.0062)	0.6066 (0.0010)	0.6121 (0.0007)
$\sigma_{\eta_{12}}$	0.9837 (0.0071)	0.6844 (0.0012)	0.5916 (0.0008)
$\sigma_{\eta_{13}}$	1.0785 (0.0082)	0.7810 (0.0015)	0.5710 (0.0008)
$\sigma_{\eta_{14}}$	1.1845 (0.0095)	0.9014 (0.0019)	0.5505 (0.0009)
$\sigma_{\eta_{15}}$	1.3031 (0.0109)	1.0521 (0.0025)	0.5301 (0.0009)
Simulation draws:[a]	50	50	50

[a]Number of simulation draws used in calculating $v_n(x, x; \theta)$

Table 9: Contract-specific covariates: Highway-work contracts

Variable	N	Mean	Std Dev	Minimum	Maximum
Total Auctions	415				
NY[a]	410	7588.57	737.28	6234.50	8967.49
PHIL[b]	410	6005.94	521.88	5083.90	7311.37
TRAFFIC[c]	364	27996.41	21227.55	0	97235.00
GATEWAY[d]	364	0.56	0.50	0	1.00
SKYLANDS[e]	364	0.35	0.48	0	1.00
SHORE[f]	364	0.24	0.43	0	1.00
DELAWARE[g]	364	0.49	0.50	0	1.00
SOUTH[h]	364	0.26	0.44	0	1.00

[a]Construction Cost Index (CCI) for New York area, in 1913$. Source: ENR (1990-1997).

[b]Construction Cost Index (CCI) for Philadelphia area, in 1913$. Source: ENR (1990-1997).

[c]Weekday traffic volume of road being repaired, in both directions. (Source: NJDOT)

[d]GATEWAY=1 if road lies (partly) in Bergen, Hudson, Middlesex, Passaic, Union, Essex counties.

[e]SKYLANDS=1 if road lies (partly) in Hunterdon, Morris, Somerset, Sussex, Warren counties.

[f]SHORE=1 if road lies (partly) in Ocean or Monmouth counties.

[g]DELAWARE=1 if road lies (partly) in Camden, Gloucestor, Salem, Burlington, Mercer counties.

[h]SOUTH=1 if road lies (partly) in Atlantic, Cumberland, Cape May counties.

Table 10: Parameter estimates: accounting for observed contract heterogeneity
Standard errors in parentheses.
Estimated for worktype 4 contracts only.

Parameter:	
σ_w	0.2767 (0.0014)
σ_a	0.9023 (0.0004)
σ_e	2.3400 (0.0072)
μ_0	-2.0214 (0.0088)
μ_1	0.4179 (0.0072)
μ_2	-0.0545 (0.0015)
COST[a] (μ_3)	3.469E-5 (1.761E-7)
TRAFFIC (μ_4)	-4.856E-7 (2.331E-9)
GATEWAY (μ_5)	0.0616 (2.399E-4)
SKYLANDS (μ_6)	0.0760 (7.369E-4)
SHORE (μ_7)	-0.0062 (4.424E-5)
DELAWARE (μ_8)	-0.0190 (6.495E-5)
SOUTH (μ_9)	0.1479 (5.694E-4)
#contracts	356
Simulation draws:	
R_2[b]	100

[a]For definitions of covariates, see table 9. COST $= \frac{1}{2}(\text{NY} + \text{PHIL})$.
[b]Number of simulation draws used in calculating $v_n(x, x; \theta)$