The Johns Hopkins University
Department of Economics
Microeconomics Comprehensive Examination
June 6, 2011

Instructions:

- You have 4 hours for the exam plus an additional 15 minutes to read it.
- Please answer all questions in Parts A and B, and two out of three questions in Part C.
- **PLEASE MAKE SURE TO DEFINE YOUR TERMS PRECISELY INCLUDING ANY NOTATION THAT YOU INTRODUCE.**
- **PLEASE START ANSWERING EACH QUESTION ON A NEW PAGE.**

Part A

1. **20 points**

   (a) Explain concisely and precisely the differences between the concepts listed under each category.

   i. The convexity of the constraint set, the Slater's form of the constraint qualification and the concavity of the functions specifying the constraints; all of these in the context of the theory of constrained optimization.

   ii. A cost function and an expenditure function.

   iii. The duality and equilibrium theorems of linear programming.

   (b) Versions of separating hyperplane theorems and fixed point theorems assert the existence of a price system, to use the terminology of economic applications. As instruments for the development of economic theory, are they *substitutes* or *complements?* Give reasons for your answer, and include precise definitions of the italicized terms.

2. **20 points**

   (a) Define correlated equilibrium.

   (b) True of false?

   If a strategy profile is a Nash equilibrium, then it is also a correlated equilibrium.

   If true, then prove the statement. Otherwise, provide a counterexample.

3. **20 points**

   Consider a preference relation on $\mathbb{R}^n_+$ satisfying the axioms Weak Order, Archimedean and Independence.

   (a) Characterize the representation of this preference relation.

   (b) Does the representation make sense? Explain

Part B

4. **30 points**

   Let $\{f(x|y)\}_{y\in[0,1]}$ be a family of conditional density functions of a random variable, x, taking values in the interval $[a,b]$. Let $\{F(.|y)\}_{y\in[0,1]}$ be the corresponding conditional cumulative distribution functions. Prove that if this family satisfies the monotone likelihood ratio property then $F(.|y')$ first order stochastically dominates $F(.|y)$, for all $y' > y$. (In your answer, make sure to define the terms monotone likelihood ratio property and first order stochastic dominance).

5. **30 points**

   Consider a planning problem in which a unit amount of perfectly divisible labor is available each (discrete) time period to be allocated between an investment and a consumption sector. As regards the former, labor can be allocated to produce one or many types of a possible finite number (say m) types of machines. A machine of type i requires $a_i > 0$ units of labor to construct it. As regards the consumption sector, a unit of labor can be used with a unit of machine of type i, if available, to produce $b_i > 0$ units of an identical consumption good. Each type of machine is also perfectly divisible, which is to say, can be used in fractional amounts, and depreciates at an annual percentage rate of d.

   (a) In terms of two-dimensional diagrams, represent the technology available to the planner. Briefly note the difference between an *isoquant,* a *production function* and a *transformation function,* (also referred to as the *production possibility curve).* What basic economic principle underlies each of these three constructs?

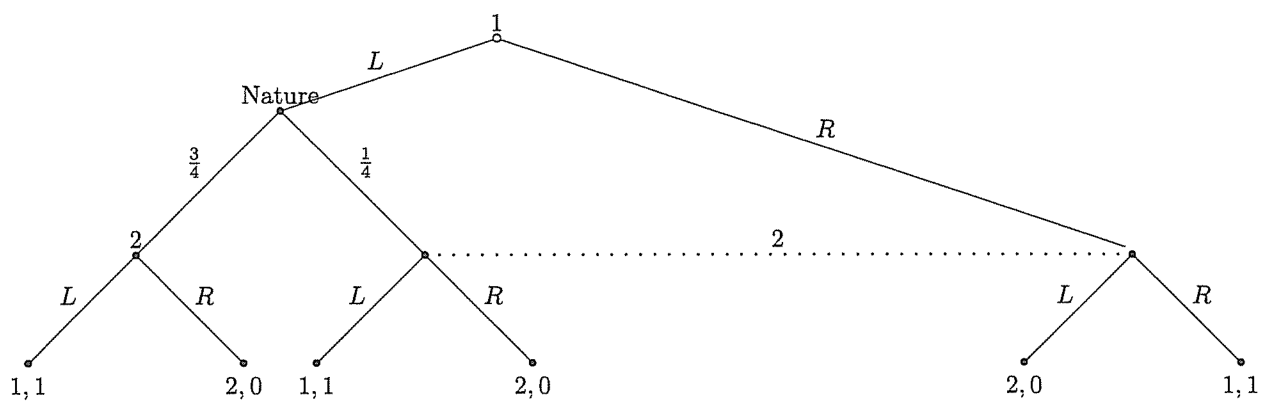
   (b) Indicate how the technology available to the planner at any point in time is a special instance of the "two-sector general equilibrium model"? Does Rybczynski's theorem apply to this specialization. If so, indicate how? If not, indicate why not?

   (c) If the planner's objective is to maximize the output of the consumption good that remains constant over time, what type, or types, of machine should he build, and what are their amount or amounts?

   (d) Provide an interpretation of your answer in terms of a theory of value based on labor.

   (e) Suppose the planner is interested in maximizing (i) a discounted (ii) undiscounted sum of the consumption good that is feasible given a particular initial stock of machines. In each case, develop precise conjectures on how the planner should allocate labor in each period, and justify them.

6. **30 points** Consider the following game:



   (a) What is the pure strategy set for each player?

(b) What is the mixed strategy set for each player?

(c) What is the behavioral strategy set for each player?

(d) Define weak sequential equilibrium for this game.

(e) Find the weak sequential equilibria.

Part C

7. **50 points**

Consider a take-it-or-leave-it bargaining game between two players over two issues. Issue one is a cake division problem, that is, players decide how to split \$1, and the set of feasible divisions of the cake is given by $X = \{(x_1, x_2) \in [0,1]^2 : x_1 + x_2 \leq 1\}$. Issue two is a policy decision, that is players choose a policy $y \in [0,1]$. Payoff of player i over the final outcome (x_1, x_2, y) is given by

$$u_i(x_1, x_2, y) = x_i - \frac{1}{2}(y - \hat{y}_i)^2$$

where x_i denotes player i's cake share under the final outcome and $\hat{y}_i$ is the ideal policy of player i.

The rules of the game is as follows. Player 1 makes a take-it-or-leave-it offer (x_1, x_2, y) to player 2. If player 2 accepts, the game ends and the offer is implemented. If player 2 rejects, the game ends with the policy outcome $\tilde{y} \in [0,1]$ and neither player receives any cake. Assume that $\hat{y}_1 > 2\hat{y}_2 > \tilde{y} = 0$.

(a) Define Nash equilibrium for this game.

(b) Define subgame perfect Nash equilibrium for this game.

(c) Characterize the subgame perfect Nash equilibria.

8. **50 points**

(a) The *equal treatment property* is informally stated as a situation in which *identicals are treated identically.* In the context of a pure exchange economy, give a precise and formal statement of this intuition.

(b) Define the notion of a *competitive equilibrium* for a pure exchange economy. Does this solution concept exhibit the *equal treatment property?* Justify your answer in either case. Do the properties of preferences have any relevance for your answer?

(c) Define the notion of a Pareto optimal allocation, and indicate whether it exhibits the *equal treatment property?* If so, provide a proof. If not, develop an argument as to why we should have any interest in this concept.

(d) Define the notion of a *core* for a pure exchange economy. Does this solution concept exhibit the *equal treatment property* in the context of a pure exchange economy which is replicated in the sense that there are an equal number of agents of a finite number of types? Justify your answer in either case. You may assume that the preferences of each type of agent are strictly convex.

(e) Indicate how your answer to 7d above is modified in the context of exchange economies which do not necessarily have an equal number of agents of a finite number of types.

9. **50 points**

Consider the double-auction mechanism involving a buyer and a seller. The buyer's valuation (type) of the object, v_b, is a random draw form a uniform distribution on $[0, 1]$, and the seller's valuation is $v_s = 0$.

According to the mechanism the buyer and the seller submit sealed bids, β_b, and β_s, respectivley. If $\beta_b < \beta_s$, then the seller keeps the object and no monetary transfer is made, and if $\beta_s \leq \beta_b$, the buyer gets the object and the buyer pays the seller the amount β_s.

Suppose that the buyer's utility function is $u_b(k, \beta_s) = (v_b - \beta_s)\, k$, where $k = 1$ if the buyer ends up possessing the object and $k = 0$ otherwise. The seller's utility is $u_s(k, \beta_s) = k\beta_s$.

(a) Solve the Bayesian-Nash equilibrium of the game induced by this mechanism.

(b) What is the seller's expected utility in this mechanism?

(c) Is the outcome ex-post efficient? Explain

(d) Is it possible to implement using this mechanism a social choice function that is Paretian? Explain.